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The impact of taking a college pre-calculus course on students' college calculus performance

Gerhard Sonnert* and Philip M. Sadler

Harvard–Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

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Poor performance on placement exams keeps many US students who pursue a STEM (science, technology, engineering, mathematics) career from enrolling directly in college calculus. Instead, they must take a pre-calculus course that aims to better prepare them for later calculus coursework. In the USA, enrollment in pre-calculus courses in two- and four-year colleges continues to grow, and these courses are well-populated with students who already took pre-calculus in high school. We examine student performance in college calculus, using regression discontinuity to estimate the effects of taking college pre-calculus or not, in a national US sample of 5507 students at 132 institutions. We find that students who take college pre-calculus do not earn higher calculus grades.

Keywords: college pre-calculus; college calculus; performance

1. Introduction

At first sight, American college-bound high school graduates have grown in mathematical sophistication over the past several decades. The percentage of students who graduated high school having taken pre-calculus more than doubled from 1990 to 2009 to 35%. Calculus was taken by 17% of high school students in 2009, up from 7% in 1990.[1] Whereas in the 1950s, few students entered college with experience in college-level coursework, nowadays a high school calculus class – or an Advanced Placement (AP) calculus class, or a high AP calculus exam score – has become something that is highly recommended and seen as generating a competitive edge for students intending to get into an elite college or university.[2] About 600,000 high school students study calculus in some form each year.[3]

With a closer look, this rather idyllic picture of a generation of math-savvy American students reveals serious blemishes when they arrive in college. Concerned observers have noted that incoming students' mathematics preparation is often below par, and that weak algebra and pre-calculus skills detrimentally impact students taking calculus,[4] which in turn does not bode well for career aspirations in science and technology. Reflecting these mathematics deficits, the numbers of students taking mathematics classes below the calculus level in college are large and growing. At two-year colleges, the number of students enrolled in 'precollege' (i.e. 'remedial') and pre-calculus (introductory) level classes rose by a third, from 969,000 to 1,285,000, between 1990 and 2005, while, over the same period, the number of students enrolled in calculus level classes declined from 128,000 to 107,000 [5, p.140, Table TYE.4] (Numbers do not include dual enrollments.) Apparently,

*Corresponding author. Email: gsonnert@cfa.harvard.edu

substantial and rising numbers of students are deemed or feel so ill-prepared for college calculus courses that they choose, or are advised to take, or are placed in, college classes below the calculus level.

Though the issue may be most salient at two-year colleges, at four-year institutions, too, students taking calculus are clearly outnumbered by those taking lower level mathematics courses. At that type of institution, the number of students enrolled in precollege (remedial) and pre-calculus level classes increased from 835,000 to 909,000 between 1990 and 2005, while the group of calculus enrollees grew, at a lower level, from 538,000 to 587,000.[5, p.82, Table E.2][1]

Only 30%–40% of the students who pass college pre-calculus courses ever start Calculus I.[7] A study at Texas Tech [8] found that of the 'successful' (i.e. receiving the grade of B or above) students in the analytical geometry course, 67% enrolled in Calculus I, but only 35% were successful (by the same measure) in Calculus I.

As the number of college students in the lower level courses has risen, these courses have become an important part of the ecology of mathematics education in American colleges. In our own study of a national sample of American college calculus takers, 31.9% had previously enrolled in a college pre-calculus course. Of these college pre-calculus students, more than half had taken and passed a pre-calculus course while in high school (with nearly half earning the grade of A). Nearly a fifth had taken and passed a high school calculus course (with more than one-third earning the grade of A).

Commentators have decried the apparent deficiency of high school mathematics preparation as the root cause of college students being poorly prepared to take college calculus, even with high school pre-calculus or calculus taken previously.[4,9] Many have called for reform in secondary mathematics instruction. For instance, in 2000, the National Commission on Mathematics and Science Teaching for the 21st Century issued its evocatively titled report *Before It's Too Late*, criticizing the current state of mathematics and science education in the USA and urging improvements.[10] In mathematics, professional associations, such as the National Council of Teachers of Mathematics (NCTM), have been spearheading curricular reform,[11] and, in 2010, the *Common Core State Standards for Mathematics* were released by the Council of Chief State School Officers and the National Governors Association Center for Best Practices.[2] In the meantime, however, while such high school efforts are under way and will, one hopes, lead to progress down the road, colleges and universities need to deal with the cohorts of students who arrive at their doors right now.

For the foreseeable future, then, remedial and preparatory mathematics classes are likely to remain an important fixture of mathematics teaching at American institutions of higher education. These classes are certainly not the most glamorous for the tenured or tenure-track mathematics faculty to teach, and thus prime candidates for being entrusted to adjuncts or graduate assistants, or being taught outside of the mathematics programme, e.g. in developmental studies divisions.[12] Even though college courses that are designed to prepare students to undertake calculus in college have received some attention by educational policy-makers (e.g. [13,14]), it is probably fair to say that they have taken the backseat vis-à-vis the – of course also important – focus on calculus reform and other issues. Similarly, the community of educational researchers has not made work on this topic a priority. As a consequence, in 2008, Calcagno and Long [15,p.i] rightly bemoan that 'despite its important role in higher education and its substantial costs, there is little rigorous evidence on the effectiveness of college remediation on the outcomes of students'.

This article intends to shine a spotlight on one part of this somewhat neglected area, college pre-calculus.

2. Literature review

The issue of 'remedial' or 'developmental' courses at the college level is not unique to mathematics, but much more pervasive, vexing American higher education in general (e.g. [16]). These courses represent content that is taught in American high schools, but with which many incoming college students are found unfamiliar. In the fall of 2000, 28% of entering college freshmen took one or more remedial reading, writing, or mathematics courses.[17] Whereas there are occasional attempts to push the problem back into the high schools, the reality is that these courses are an integral part of American higher education – and have been for a long time. We should caution that the 'good old days' when every student arrived well-prepared in college are simply a figment of nostalgic imagination. As Merisotis and Phipps [18] pointed out, American institutions of higher education have chronically faced underprepared students. Nonetheless, the problem is particularly pressing today, and mathematics is one of the major focal points in terms of subject area. In institutional terms, the main battleground is community colleges, where the philosophy of 'open access' collides head-on with the reality of student under-preparedness. Community colleges are forced to offer a wide array of remedial and developmental courses that are not classically considered college-level, especially in mathematics.

The pervasiveness of developmental courses in institutions of higher education brings at least two issues to the fore: How efficient are the placement tests that are commonly used to sort students into different levels of knowledge and assign them to different courses, from developmental to advanced? And, how efficient are the developmental courses themselves in preparing students for success in college-level coursework?

Because of the increased role that placement tests play for charting students' trajectories through higher education, and because of the increased educational and economic stakes associated with the tests, a main focus of research has been on the first of these issues, on the validity of placement tests. When reviewing predictive validity studies conducted before 2000, Armstrong [19] found that these studies yielded only low correlation coefficients between placement test score and final grade. His own study of mathematics and English courses in three large community colleges showed that, even though placement test scores and course grades were somewhat correlated, 'the coefficients were too low to be of much practical significance'.[19,p.688]

While it is relatively straightforward to calculate correlations between placement test scores and course grades, the appropriateness of such a measure to evaluate placement tests is questionable, as pointed out by Armstrong [19] and McFate and Olmsted [20], among others. To predict course grades is not the purpose of placement tests. Rather, their primary function is to separate those students who are not prepared for taking a certain college course from those who are. With an ideal placement test, all students above the cut-off score would succeed in that course, and all below would fail. Apart from that, it should not really matter if placement test scores predict course scores or not.

Mattern and Packman [21] of the College Board conducted a meta-analysis to determine the validity of placement decisions made on the basis of the College Board's ACCUPLACER® test. This meta-analysis comprised 47 studies at 17 institutions, out of which 14 were two-year community colleges. Instead of using correlations of placement test grades with course grades, they used more pertinent dichotomous outcome measures of 'success' in the course, which was defined in two ways – the grade of B or higher, or the grade of C or higher. The raw correlations were smallish (0.24–0.28), but after correction for various statistical artefacts, the correlations became stronger (0.38–0.47). Using similar success measures in her study of more than 42,000 students in a large, urban community college system, Scott-Clayton [22] found that overall the correlation between placement exam scores and later course outcomes was relatively weak. She also found that placement exams were more predictive of success in mathematics than in English.

Analysing data from a statewide community college system, Belfield and Crosta [23] found that standard placement tests (ACCUPLACER® and COMPASS® in both mathematics and literacy) did not strongly predict how students would perform in college (measured as college grade point average [GPA]), and that the correlation totally disappeared when controlling for high school GPA. The authors did find that placement test scores were positively associated with students' college credit accumulation, even after controlling for high school GPA. In concurrence with Scott-Clayton's [22] results mentioned above, they also estimated that the placement error rates were higher in English than in mathematics, though the latter rates were still disconcertingly high, impacting around a quarter of students. Overall, Belfield and Crosta [23] found high school GPA to be a superior predictor than the two placement tests.

Whereas correlating placement tests with a measure of success in a course other than the scale of grades received constitutes a methodological advance, another problem still exists. That difficulty in determining the quality of placement tests lies in the fact that only one of the two possible placement errors is easily observable: the 'over-placement error', in which students do badly in the course in which they were placed. By contrast, the 'under-placement error', in which students are placed in a lower level course even though they could have succeeded in the higher level course, eludes direct measurement and must be estimated. Of course, a random assignment of students to courses following the taking of a placement test might resolve this, but both pragmatic and ethical issues would make conducting such experiments problematic.

The second issue is connected to the topic of placement tests, but perhaps even more fundamental: What is the effect of taking a pre-calculus course on subsequent performance in a calculus course, i.e. did the students reap a benefit from their pre-calculus participation and, if so, how large was it?

Research findings are mixed for the effects of developmental classes in English. In a large, multi-campus community college, Moss and Yeaton [24] found positive effects of English remedial classes on academic achievement (in terms of grades in students' first college level English course). By contrast, on a different community college campus, Horn, McCoy, Campbell, and Brook [25] found that participation in a developmental programme was negatively associated with performance in a subsequent English class.

As to the situation in mathematics, Harrell and Lakins [26] examined how well students at a small liberal arts college who had attended lower level mathematics courses did in subsequent courses. Among their findings was that students who had opted out of the remedial (intermediate algebra) mathematics course, Math A, '(presumably because they felt, despite placement test results, that they were already adequately prepared) often fared [in subsequent courses] as well as, or better than, students who placed in Math A and took it first'.[26,p.38] On the other hand, in a study at another four-year institution, Lesik [27] found that developmental mathematics courses reduced student dropout.

While many studies on the topic cover single institutions (with the research conducted by faculty members at these very institutions), or at most cover only small numbers of institutions, studies of a larger scale are rare, but a few exist. In a study of nearly 100,000 community college students in Florida, Calcagno and Long [15,28] examined the causal effects of remediation in mathematics and reading on educational outcomes. The researchers essentially found no benefit of remediation on the completion of college-level credits or eventual degree completion. Furthermore, two state-wide studies in Texas on the effects of developmental courses on students' outcomes, by Martorell and McFarlin [29] and Miller [30], were rather disheartening. Neither study found that the college students who were assigned to developmental courses, based on their score on assessment tests, benefitted from these courses. On the contrary, their results suggested that these developmental courses actually had a negative impact.

None of these larger studies addressed the issue of pre-calculus courses specifically. Whereas any lower level mathematics course that is sequenced before (pre) calculus might be considered 'pre-calculus' in a general sense (e.g. also an algebra or even arithmetic class), we use a narrower definition of pre-calculus as the *last* course before calculus.

Many argue that the proper measure of the success of a course is the students' performance in the next course in the sequence.[8,31,32] Because one of the purposes of college pre-calculus courses should obviously be to prepare students for their successful participation in a college calculus course, our research question is: To what degree, if any, do college pre-calculus courses actually improve students' performance in a subsequent college calculus course?

3. Data and methods

The data used in this article, from the 'Factors Influencing College Success in Mathematics' (FICSMath) study that was conducted at the Science Education Department of the Harvard-Smithsonian Center for Astrophysics, were collected through a questionnaire containing 61 questions that was administered in the fall semester of 2009. We obtained a stratified national sample of 10,437 students enrolled in 336 college calculus courses/sections at 134 institutions. In addition to students' grades in college calculus, extensive information was collected about their demographic background and their prior mathematics education.

For our sample selection, the distinction between four-year and two-year institutions served as the first stratification criterion. Each of the two groups thus obtained was further stratified by the size of the institution (small, medium, and large). The National Center for Education Statistics (NCES) kindly transmitted, from the Integrated Postsecondary Education Data System (IPEDS), a table of 4305 degree-granting postsecondary institutions in the USA, which included 1668 two-year and 2637 four-year schools. Additionally, the table included fall 2007 enrollment numbers for two-year institutions and fall 2006 enrollment numbers for four-year institutions. Using the institutions' overall undergraduate enrollment numbers (both full-time and part-time undergraduates), it was determined that roughly a third of the national undergraduate population attended schools that had fewer than 5400 undergraduates (termed 'small' institutions), another third attended schools that had between 5400 and 14,800 undergraduates (medium), and the final third attended schools with more than 14,800 undergraduates (large). The cut-offs were similar for four-year and two-year institutions. The 4305 institutions listed in the NCES table were thus stratified by type and size into six lists: 2089 small four-year colleges, 348 medium four-year colleges, 200 large four-year colleges, 1279 small two-year colleges, 289 medium two-year colleges, and 100 large two-year colleges.

Each of these six lists of institutions was randomized. We then recruited institutions by going down each list until we had enough positive responses from institutions that, in our estimation, a sufficient number of students in the respective category could be reached. The recruitment process itself served as a crude gauge of the distribution of students across the categories. We extrapolated the projected numbers of calculus students from the institutions that agreed to participate, as well as from the institutions that we found to have no calculus offerings. This extrapolation suggested that about a quarter of the national calculus student population attended two-year schools, and three-quarters attended four-year schools. This division is roughly mirrored in our sample, which contains 33.2% students from two-year schools and 66.8% students from four-year schools. It also became obvious that small schools rarely offer calculus. Only an extrapolated 2% and 7% of the calculus takers went to small two-year and four-year institutions, respectively. Correspondingly, 1.8% of students in our sample were at small two-year schools and 8.3% attended small four-year schools. The bulk of the students taking calculus went to medium and large institutions. The sample proportions track the rough extrapolations reasonably well, with the possible exception that the sample may contain a lower percentage of students in medium-size four-year schools, compared with the percentage of those students in the population (according to our extrapolation): 23.0% vs. 39.9%.

Of the 276 institutions contacted, 182 (65.9%) initially agreed to participate. In the end, we received usable student questionnaires from 134 (i.e. 73.6% of those who agreed to participate or 48.6% of all contacted institutions). The questionnaires were administered in class, which was conducive to a very high student participation rate in each classroom that took part in this study.

The FICSMath survey was pilot tested with 47 students at two local institutions. Pilot testing also established that filling out the questionnaire took on average 15–20 minutes. To ensure the validity of the questions asked, the construction of the survey was guided by open-ended online surveys in which high school mathematics teachers and mathematics professors were asked about what teachers did – or should do – to prepare students for success in college calculus. In addition, a focus group with experts in science and mathematics education was held that confirmed the validity of the survey. To gauge reliability, we conducted a separate test–retest study in which 174 students from three different colleges took the survey twice, two weeks apart. Our analysis found that, for groups of 100, less than a 0.04% chance of reversal existed.[33,p.117]

3.1. *Methodological approach*

A perfect measurement of the effects on subsequent calculus performance of taking a college pre-calculus course is elusive. True random-assignment experiments that could provide definitive answers have not been conducted, and carrying them out would be difficult, if not impossible, for obvious reasons. Hence, researchers must rely on various sorts of correlational methods. When, in a correlational framework, one compares the college calculus performance of students who had previously taken college pre-calculus with that of students who had not taken such a course, one is immediately confronted with the question: Is any observed difference in calculus performance really *caused* by the students' prior taking or not-taking pre-calculus? Or are these differences rather caused by other student characteristics that influence both students' performance in calculus and their taking college pre-calculus? Conversely, if the students with prior pre-calculus were found to perform no better in calculus than students who had not taken college pre-calculus, this would not prove that the pre-calculus course was ineffective. A defender of the effectiveness of college pre-calculus might well argue that those students who took the college pre-calculus class came to college with a particularly weak mathematics preparation, and that they would have done *even worse* without that pre-calculus course. For instance, if there was no significant difference in college calculus grade between students who did not take college pre-calculus, but had taken calculus in high school, and students who took college pre-calculus without having had high school calculus, one could plausibly suggest that college pre-calculus might have a positive effect (of bringing the initially less well prepared students up to par).

One possible strategy to assess the efficacy of a college pre-calculus course is statistically to account for differences in student background (e.g. high school mathematics courses taken, SAT/ACT mathematics test scores, parental education, etc.) in a regression model that contains these background variables as controls. However, this strategy faces the problem of collinearities between the control variables and the variable of interest (e.g. students with low SAT/ACT scores are more likely than those with high scores to take college pre-calculus). A more sophisticated approach is to eliminate the collinearities by equalizing the samples of pre-calculus takers and non-takers on as many relevant student characteristics as possible. The statistical method of choice to accomplish such an equalization is propensity weighting, which in this case would produce two groups (pre-calculus takers and non-takers) that have a very similar mathematics preparation (in terms of SAT mathematics scores, mathematics course grades, etc.) and very similar general background characteristics (in terms of gender, race, parental socioeconomic status, etc.), on average. Propensity weighting achieves this by, for instance, giving high weights to the few students with a strong mathematics background who happened to take pre-calculus, and to the students who did not take pre-calculus even though their preparation was weak. Then, the central argument of the propensity weighting approach is that any differences in the weighted groups' subsequent calculus performance can no longer be attributed to those student characteristics that had been levelled. A stronger case could thus be made that any observed differences in calculus performance are due to the fact of taking or not taking the pre-calculus course.

Our strategy is different. It does start with a propensity analysis, but then makes a U-turn and pursues a purpose diametrically opposed to the conventional propensity weighting objective. We use propensity not to equalize, but to completely separate groups of students with the goal of using these groups for a discontinuity analysis. This strategy is based on the recognition that the college pre-calculus course has a specific purpose and a specific target population. One of its main goals is to help weaker students get ready for calculus; it is not meant for all students and certainly not for students arriving with a strong mathematics background. It seems therefore problematic to statistically equalize the background of pre-calculus takers and non-takers. The groups thus created (through propensity weighting) are somewhat perverse because they give overproportionate weight to those students who should *not* have been in the respective groups in the first place. Hence, it is unclear what a comparison of these two groups would mean.

In this article, we draw on students' characteristics that indicate the strength of their mathematics preparation to separate them into simulated groups of pre-calculus takers and non-takers that can be analysed by regression discontinuity to study the effect of taking pre-calculus. Regression discontinuity has recently become a method of choice for researchers investigating the effects of taking remedial and pre-calculus classes.[15,24,25,27–29]

Our strategy is to *simulate a forced assignment* of students to either group, based on their mathematics preparation (modelled by their propensity score). Imagine college officials who use a set formula translating the incoming students' mathematics background into a score of mathematics preparation and place all students below a certain cut-off score in compulsory pre-calculus, whereas the others, who score above, are not allowed to enroll in pre-calculus. The two groups (pre-calculus takers and non-takers) are now completely separated by the strength of their mathematics preparation. However, the measure used to separate the groups is continuous. Therefore, the students *around the cut-off* can be assumed to be of about equal preparation. (For example, a student scoring 0.51 and another student scoring 0.49 on this measure have a very similar mathematics preparation. Yet, if 0.50 is the cut-off, they are assigned to different groups.) Then, because the students on either side of the cut-off are similar in their mathematics preparation, the influence of mathematics preparation is controlled for, and any difference in calculus performance between them could, with some plausibility, be attributed to the fact of their taking or not taking the pre-calculus course.

The regression discontinuity we use to carry out this comparison consists of two parts. It estimates a first regression line of calculus performance for the pre-calculus takers (with the continuous preparation measure – the propensity score – as the predictor) and a second line for the pre-calculus non-takers. The domains of these two regressions touch right at the cut-off point. If we observe a gap, or discontinuity, between the two regression lines at the cut-off, we could interpret this ‘jump’ in predicted calculus performance as the effect of taking the pre-calculus course (for an example of a regression discontinuity, see Figure 3).

A single regression discontinuity works at one designated cut-off point. But what if there were different potential cut-off points that might yield different results? For instance, one might conjecture that pre-calculus provides benefits only for particularly weak students, i.e. at a low cut-off. We therefore estimate a whole array of regression discontinuities that simulate cut-offs ranging from very restrictive to very expansive policies of students’ forced assignments to pre-calculus. Since institutions of higher learning vary both in the kinds of measures used to assess student readiness for mathematics courses and in the cut-off scores used on these measures, we do not know the exact rules employed. Luckily, econometricians have developed methods for carrying out a regression discontinuity when the exact discontinuity point is unknown.[34,35] In our approach, we establish a reasonable measure of mathematics preparation, which is available, across institutions, for all students in our sample, as a basis for systematically varying (simulated) institutional policy.

3.2. *Filters*

The FICSMath study assembled a large sample of about 10,500 calculus students in American institutions of higher education. These students represent the great diversity of educational pathways found among the student population. Because our aim in this article is to examine the effect of taking college pre-calculus courses, we considered it advisable to make the student sample as homogeneous in certain relevant background characteristics as possible and thus to enhance comparability of the groups of pre-calculus takers and pre-calculus non-takers.

Specifically, we excluded the following FICSMath participants from this analysis:

- students who went to high school abroad or were home-schooled;
- students who had previously taken college calculus and were repeating it;
- students who had taken pre-calculus at a different college;
- students with a lapse of five or more years between the time taking the survey and taking their most advanced high school math course;
- students who did not take Algebra 1 in either eight or ninth grade;
- students who were not undergraduates (i.e. who were graduate students or ‘other’ students).

Finally, for the regression discontinuity, institutions that had fewer than three students in either category (pre-calculus takers and non-takers) were excluded.

These exclusions served to create a somewhat homogeneous and still large sample of 5507 students in 132 institutions to which the research question at hand was reasonably applicable. To prevent students with some missing data from being deleted from this sample, the method of multiple imputation [36] was used.

3.3. *A measure of high school mathematics preparation*

The incoming students' level of mathematics preparation should be the major determinant of their taking pre-calculus in college. Specifically, as mentioned, a major goal of pre-calculus courses in college is to prepare those students who might otherwise struggle in college calculus for success in that course. Colleges use a variety of often idiosyncratic strategies of placing students in pre-calculus classes, or leave the decision entirely up to the student. Some use standardized placement tests (e.g. ACCUPLACER, SAT, ACT), others use 'homemade' tests, while some use students' mathematics achievement history (e.g. type of high school mathematics courses taken, grades achieved, AP exam scores). Some colleges make placement recommendations, while others make obligatory placements.

In light of the findings by Belfield and Crosta [23], reported above, that, in some respects, high school based measures are equal if not superior to placement tests in predicting subsequent performance, it seems reasonable to construct a measure of high school mathematics preparation – which also has the advantage that it can be obtained, across the institutions, for all students in the sample. From the FICSMath survey, we have the benefit of detailed information of the students' high school mathematics preparation, which serves as input for our simulated compulsory pre-calculus placements. There are two ways of constructing a measure of high school mathematics preparation. One would be to create a composite variable of the 'usual suspects' and then, for confirmation, predict pre-calculus course taking from that composite variable in a logistic regression. A second way is to use those usual suspects to directly predict pre-calculus course taking. This can be achieved through a hierarchical logistic regression that accounts for institution-level differences in students' pre-calculus-taking patterns (by letting the intercept vary by institution). A student's propensity, i.e. the student's predicted probability (or, more precisely, the student's predicted log odds) of taking pre-calculus in college, derived from such a regression, can then serve as an indicator of preparation and as the individual student's preparation score. In this case, we need to confirm that the individual variables are represented in the regression equation in a sensible way (i.e. are significant and act in the correct direction).

Using the propensity scores optimizes our preparation measure for its purpose. Another advantage of the second method is that it resolves technical difficulties in composite variable construction that result from variables depending on each other (e.g. students who did not take an AP calculus course usually have no AP exam scores). The propensity scores were obtained by regressing participation in college pre-calculus on the following variables: students' mathematics SAT score (for those students who had ACT mathematics scores but no SAT mathematics scores, their ACT scores were mapped on to the SAT scale, using a concordance table published by the College Board [37]), participation in high school pre-calculus, grade in high school pre-calculus, participation in a non-AP high school calculus course, participation in an AP calculus course (both AB and BC), and score of AP calculus exam. The pseudo-R^2 of the hierarchical logistic regression (calculated as the squared correlation of observed and predicted values) was 26.3%.

Table 1, which presents the results of this hierarchical logistic regression, shows that all the independent variables indeed significantly predict participation in college pre-calculus. All coefficients are negative, as expected. (For instance, a higher SAT/ACT score corresponds to a lower propensity of taking college pre-calculus.) The independent variables were all standardized, with a mean of zero and a standard deviation of 1, to let us easily see

Table 1. Components of mathematics preparation composite, as derived from hierarchical logistic regression on college pre-calculus participation.

Predictor	Estimate	SE	*t*	*p*-value
SAT/ACT math score	−0.33	0.05	−6.59	<.0001
Took high school pre-calculus	−0.17	0.04	−4.55	<.0001
Grade in high school pre-calculus	−0.17	0.04	−4.13	<.0001
Took non-AP calculus in high school	−0.19	0.05	−3.76	0.0002
Took AP calculus in high school	−0.43	0.09	−4.76	<.0001
Score of AP calculus exam	−0.37	0.12	−2.99	0.0029
N	5507			
Pseudo-R^2	26.3			

Note: The pseudo-R^2 is calculated as the squared correlation between the predicted and actual values of the dependent variable.

the comparative weights of the independent variables in determining the predicted value of the dependent variable, i.e. the propensity score, which constitutes our mathematics preparation variable. This mathematics preparation variable can be interpreted as a weighted mean of the independent variables, with participation in a high school AP calculus course receiving the highest weight, and the score of the AP calculus exam and the SAT/ACT math score carrying similar weights. These then are the three major components of our mathematics preparation measure. The remaining three independent variables are minor components; they enter the mathematics preparation measure at about half the weight of the first three variables.

The obtained measure of mathematics preparation was transformed to make it more intuitive. First, the sign was reversed so that higher numbers indicate stronger mathematics preparation. Second, the measure was standardized (with a mean of zero and a standard deviation of 1), which will ease the understanding of the scenarios developed in the following section. Note that the value of zero on the measure corresponds to the average mathematics preparation of students enrolled in college calculus; a value of 1.00 denotes a 1-standard deviation *stronger* mathematics preparation within our sample of students.

Figure 1 represents the probability of taking a college pre-calculus course as a function of the students' mathematics preparation composite. The logistic curve that shows the expected probability from the logistic regression fits the actual data (small circles) quite well.

Figure 2, which displays the – fairly Gaussian – frequency distributions of mathematics preparation among college pre-calculus takers and non-takers, presents a different way of showing enrollment in college pre-calculus as a function of high school preparation. While the mean preparation scores of the two groups – college pre-calculus takers and non-takers – differ markedly, there is substantial overlap.

3.4. *Simulating compulsory pre-calculus placement*

Using our measure of student preparation, we can now simulate a situation in which every institution had a compulsory pre-calculus taking policy, according to which *all* students with a weaker preparation (i.e. those with a mathematics preparation score below a certain threshold) were required to take a pre-calculus, and *none* of the students with a stronger preparation was allowed to take it. This can be simply achieved by excluding the weaker

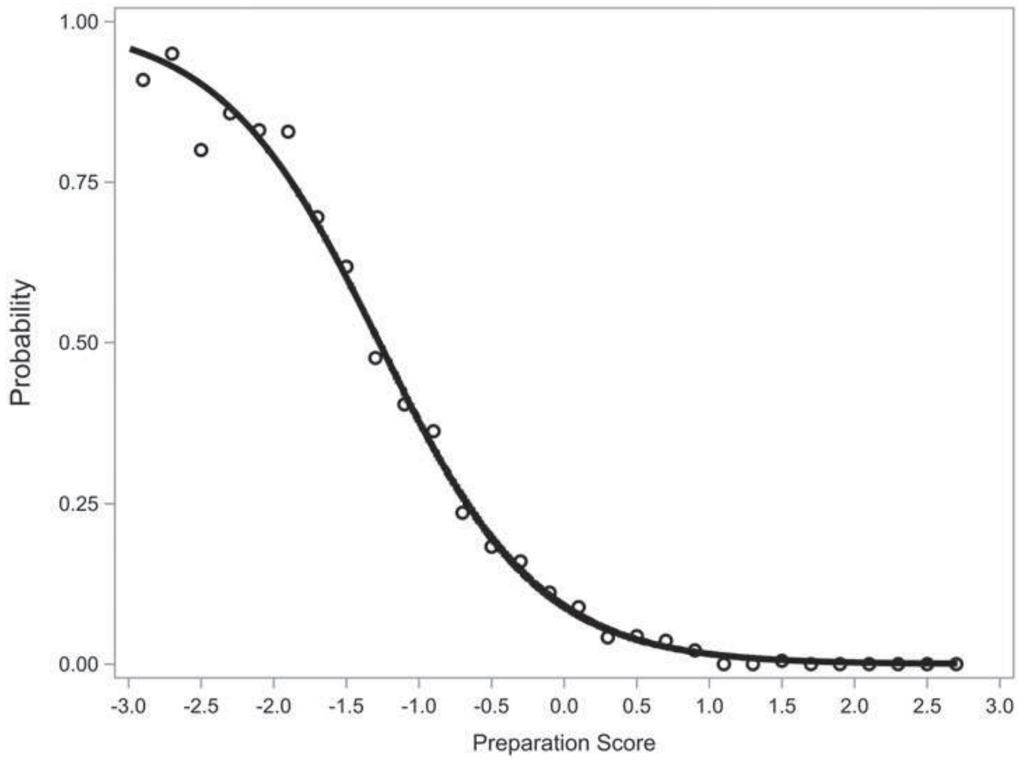

Figure 1. Probability of taking college pre-calculus as a function of students' mathematics preparation.

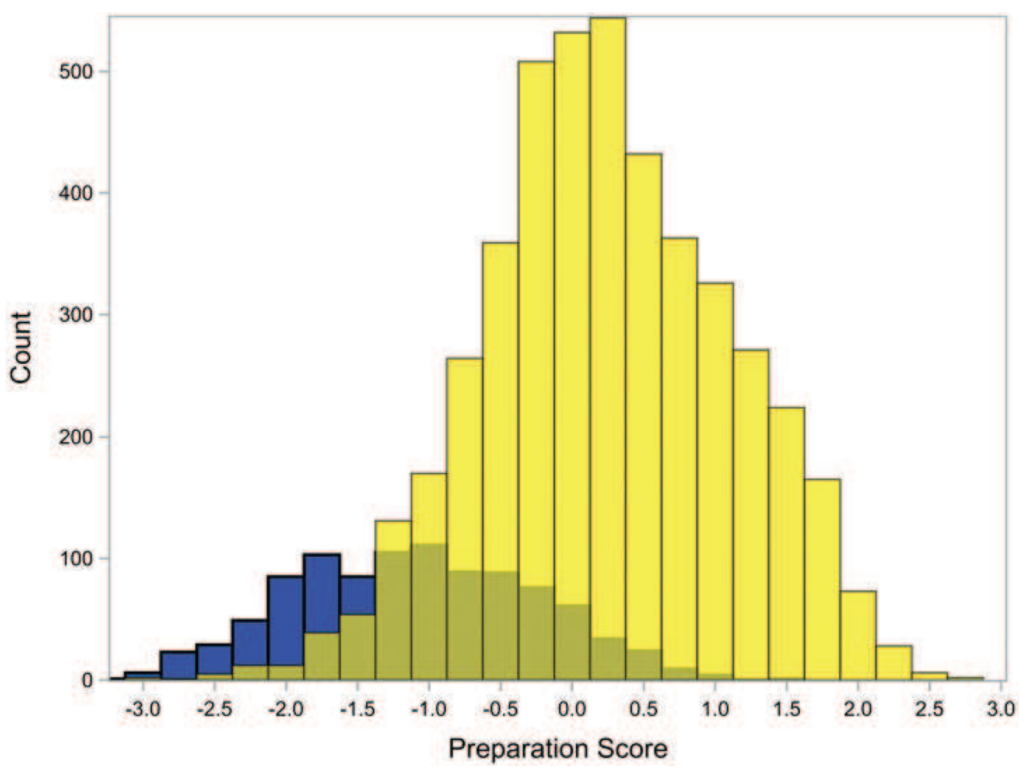

Figure 2. Histogram of mathematics preparation for students who took college pre-calculus and those who did not. Note: Blue: took college pre-calculus. Yellow: did not take college pre-calculus.

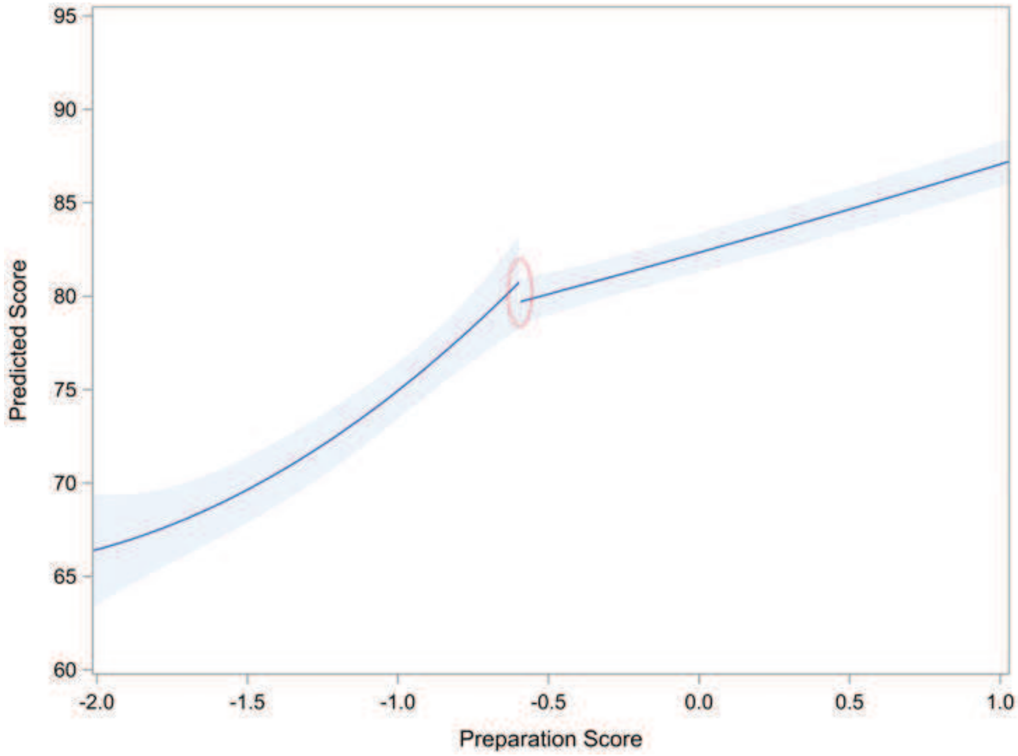

Figure 3. Regression discontinuity for college calculus grade, at a −0.6 cut-off of mathematics preparation for simulated compulsory college pre-calculus placement. Note: Shaded area indicates ±1 standard error (SE).

students without college pre-calculus and the stronger students with pre-calculus from our analysis. Figure 3 shows an example of a regression discontinuity with the cut-off for the simulated compulsory pre-calculus placement being set at −0.6 of mathematics preparation.

We then simulate various compulsory pre-calculus scenarios – from restrictive ones in which only the weakest students are placed in pre-calculus to more expansive ones in which even relatively strong students must take pre-calculus. By shifting the cut-off point on our preparation measure, we can simulate different institutional policies of obligatory pre-calculus taking and their effects. As mentioned above, the preparation measure was standardized, and our simulations sweep through the range of −1.8 to +1.0 standard deviations in increments of 0.1. Scenarios at the lower end of that range are very restrictive – only very weak students are sent to pre-calculus, while all others directly to calculus. Conversely, the scenarios at the upper end are very expansive – almost all students, except the very best, must attend pre-calculus. The range of mathematics preparation (−1.8 to +1.0) we used was the one in which one finds both types of students, i.e. those who had attended college pre-calculus and those who had not. Toward the ends of the range, the simulated scenarios become more and more eccentric, and students at the critical side of the cut-off, increasingly scarce. Above the selected range, college pre-calculus participation was very rare, while below it, it was near universal.

In each simulated compulsory placement scenario, after the 'too strong' pre-calculus students and the 'too weak' non-pre-calculus students are excluded, according to the respective cut-off scores, the remaining sample of students now becomes amenable to a

Table 2. Hierarchical linear regression models of college calculus grade, predicted by pre-calculus participation and mathematics preparation.

	Model I			Model II			Model III		
	Coefficient	SE	*p*-value	Coefficient	SE	*p*-value	Coefficient	SE	*p*-value
Intercept	80	0.5	<.0001	82.2	0.7	<.0001	82.2	0.7	<.0001
Precalc	−4.6	0.5	<.0001	−0.9	0.5	0.0956	−0.9	0.7	0.2296
Preparation				7.1	0.3	<.0001	7.1	0.3	<.0001
Precalc*Preparation							0	0.6	0.9691
N	5507			5507			5507		
Pseudo-R^2	11.4			21.6			21.6		

Note: The pseudo-R^2 is calculated as the squared correlation between the predicted and actual values of the dependent variable.

regression discontinuity. The students around the cut-off are very similar in the strength of their preparation, and the relevant difference between them is that some attended the college pre-calculus course and others did not. For each group (pre-calculus takers and non-takers) regressions are then estimated and examined for a discontinuity or gap at the cut-off. t-tests are used to determine the statistical significance of the gaps.

4. Results

A conventional regression approach (as described above) is to predict the college calculus grade for those taking college pre-calculus or not, accounting for background variables in increasingly sophisticated regression models. We used hierarchical linear models (HLMs) to also control for school effects on grades. Table 2 shows the results of this approach. Model I contains pre-calculus taking as the only predictor, with the result that pre-calculus takers have a significantly lower expected calculus grade (by 4.6 grade points) than do non-pre-calculus takers. When, in Model II, students' mathematics preparation is added as a predictor, it is highly significant in a positive direction, as expected, while pre-calculus taking is reduced to a small, and non-significant, coefficient. Model III features an interaction between mathematics preparation and pre-calculus taking, which was found to be non-significant. We further estimated models that added quadratic terms for mathematics preparation to allow for nonlinear relationships; yet none of these additions made any difference. In conclusion, then, this line of analysis shows that mathematics preparation heavily influences the college calculus grade, whereas taking college pre-calculus has no significant impact. However, as stated above, the strong collinearity of mathematics preparation with taking college pre-calculus ($r = -.51$) is worrisome. Hence, to gain a more solid methodological footing, we turn to the results of the regression discontinuity approach.

The results of our sweep of regression discontinuities that simulate forced assignments with different cut-offs of mathematics preparation are presented in Table 3 and Figure 4. Typically, we estimated HLMs with random institutional intercepts, but in two cases (indicated in Table 3), where such models proved infeasible owing to sample restrictions, we used fixed models. Because in some cases there were significant curvilinear trends, we included a quadratic term of our preparation measure in each HLM.

The table shows the estimated scores at the particular simulated cut-off for pre-calculus takers and pre-calculus non-takers. It also displays the resulting gap at the cut-off and a t-test for it. In the lower and middle ranges of cut-offs, up to 0.2 of preparation, pre-calculus

Table 3. Regression discontinuity gaps in college calculus grade through a cut-off range of simulated compulsory college pre-calculus placements.

		Pre-calculus			No pre-calculus						
Cut-off	School *N*	Student *N*	Boundary	SE	Student *N*	Boundary	SE	Gap	*t*	*df*	Significance
−1.8	12	128	72.54&	2.77	152	76.51	2.91	−3.97	−0.98912	138.5	ns
−1.7	17	167	74.04	2.62	205	78.02	2.48	−3.97	−1.10269	182.0	ns
−1.6	17	165	76.14	2.98	247	76.41	2.28	−0.27	−0.07193	187.0	ns
−1.5	18	188	78.80	2.84	240	78.81&	1.97	−0.01	−0.00318	201.2	ns
−1.4	21	221	78.03	2.71	387	77.15	1.69	0.88	0.275966	250.1	ns
−1.3	23	250	78.02	2.71	377	78.93	1.61	−0.91	−0.28892	273.0	ns
−1.2	23	258	76.32	2.60	464	78.50	1.57	−2.18	−0.71833	291.7	ns
−1.1	29	307	75.18	2.39	675	77.12	1.54	−1.94	−0.68117	364.6	ns
−1	29	264	75.89	2.58	1182	77.78	1.32	−1.89	−0.65211	313.2	ns
−0.9	28	264	75.80	2.50	1410	76.25	1.38	−0.45	−0.15613	324.6	ns
−0.8	31	271	76.42	2.40	1440	77.01	1.24	−0.59	−0.21932	325.2	ns
−0.7	31	297	79.41	2.50	1351	78.12	1.24	1.29	0.462946	349.8	ns
−0.6	29	302	80.69	2.49	1270	79.67	1.23	1.02	0.367767	353.8	ns
−0.5	29	313	77.96	2.57	1222	79.16	1.31	−1.20	−0.41727	368.2	ns
−0.4	31	333	78.87	2.27	1310	80.21	1.16	−1.34	−0.52768	392.4	ns
−0.3	33	348	81.04	2.14	1549	80.21	1.09	0.83	0.346331	413.3	ns
−0.2	32	353	81.75	2.15	1712	81.15	1.03	0.60	0.252649	413.2	ns
−0.1	30	352	83.36	2.15	1575	81.50	1.01	1.86	0.781101	408.1	ns
0	32	370	81.38	2.10	1780	80.90	1.03	0.48	0.203371	436.6	ns
0.1	30	348	81.54	2.23	1609	81.18	1.01	0.36	0.148438	400.1	ns
0.2	30	332	77.13	2.31	1644	80.78	0.93	−3.65	−1.46488	372.6	ns
0.3	28	328	76.53	2.46	1483	82.37	0.92	−5.84	−2.22536	361.4	*
0.4	27	332	76.20	2.78	1309	83.18	0.95	−6.98	−2.372	358.9	*
0.5	25	326	77.51	2.95	1175	83.29	1.02	−5.78	−1.85546	352.0	ns
0.6	23	316	77.20	3.17	1039	83.71	0.99	−6.52	−1.96102	335.8	ns
0.7	21	305	76.07	3.35	1061	83.73	0.97	−7.66	−2.19528	321.9	*
0.8	19	299	74.86	3.57	965	84.21	0.97	−9.35	−2.52737	312.7	*
0.9	19	301	74.09	3.68	862	84.03	1.04	−9.93	−2.59473	315.2	*
1	18	300	75.18	3.94	757	83.80	1.04	−8.62	−2.11597	311.2	*

Notes: All boundary estimates are from random intercept HLMs, with the exception of two fixed models marked "&". Significance level: $^{*}p < 0.05$.

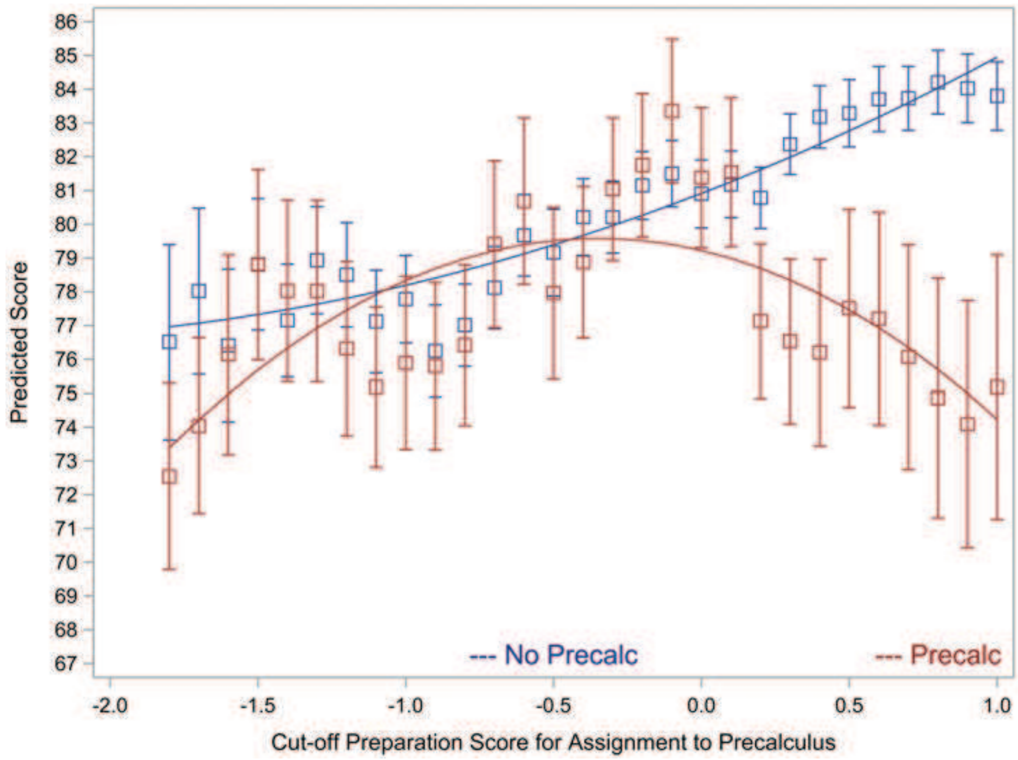

Figure 4. Regression discontinuity gaps in college calculus grade through the range of cut-offs of mathematics preparation for simulated compulsory college pre-calculus placement. Notes: Error bars indicate ±1 SE. Quadratic regression lines for the cut-off scores of college pre-calculus takers and non-takers are shown.

participation had essentially no effect at all on subsequent calculus performance. The mean calculus scores at the cut-offs were similar for pre-calculus takers and non-takers, and the differences were not statistically significant. Beginning with a preparation strength of 0.3, the pre-calculus takers actually did *worse* than the non-takers – though, because of the dearth of well-prepared pre-calculus takers, these scenarios become increasingly unrealistic, the higher one goes. (At 0.5 and 0.6, the negative gap, though still sizeable, just dropped below the 0.05 significance level.)

Figure 4 graphically represents the discontinuity gaps across the range of cut-off scores in mathematics preparation. While Figure 3 shows one regression discontinuity with its gap at a specific cut-off (−0.6), one could think of Figure 4 as composed of the gaps from 29 regression discontinuities, with the cut-off sweeping through the range at 0.1 intervals. The figure also includes trend lines for the students with and without college pre-calculus.

Even weakly prepared students did not appear to reap a benefit (in terms of subsequent calculus performance) from attending a pre-calculus class in college. At the very bottom end (−1.5), we note that the pre-calculus takers did have an advantage over the non-takers of 5.3 points, although that difference was not statistically significant. One might perhaps use this gap to speculate that college pre-calculus is effective for a group of extremely unprepared students (at −1.5 and below), but we did not have the range to investigate this further. In the low-average range of mathematics preparation (from −0.7 to 0.1), there is perhaps a hint of a benefit (although not significant) from taking college pre-calculus of about 1 point when later taking college calculus. Whether this is worth an entire semester of what is often a review of material students learned (or were supposed to learn) in high school is doubtful.

5. Limitations

As discussed earlier, this is not an experimental study of the effect of taking college pre-calculus. The regression discontinuity approach applied here is relatively advanced in terms of conducting what is sometimes called quasi-experiments. Yet, it still cannot prove causality. Other factors, not observed in our study, may be in play.

Another limitation was posed by the sample size. Although it would have been desirable to examine the effects of pre-calculus participation on calculus performance for each institution separately to determine potential differences from one institution to the other, the available number of students made this impractical. (Speaking more technically, we were precluded, in our HLMs, from letting the slope vary by institution in addition to varying the intercept.)

We should further note that the outcome we are concerned with here is not the only possible relevant outcome of taking college pre-calculus. An alternative measure of pre-calculus effectiveness may focus on the potential influence on the students' educational trajectory. For instance, there may be students who, for whatever reason, are not willing to take a calculus course directly even if they were permitted to, and for them, the possibility of taking a pre-calculus course might keep open certain STEM (science, technology, engineering, mathematics) career trajectories that might otherwise be closed. Our survey did not allow us to address these questions.

Some might consider it another potential limitation that our sample is subject to a self-selection effect. It very probably contains the more successful participants of pre-calculus courses – those who actually passed and went on to a calculus course. (As noted earlier, the majority of college pre-calculus students never make it into a college calculus course.) However, such a selection bias toward the more successful pre-calculus participants suggests that the true average calculus performance of the entire pre-calculus group would, if anything, be even weaker than the one observed. Moreover, the technique of regression discontinuity mitigates this problem because it focuses on the cut-off region of mathematics preparation.

In a similar vein, one might mention a possible general maturation effect: students becoming more experienced and efficient as they progress through their educational career. Such an effect, if it exists here, again would favour the students who took a 'detour' to pre-calculus before enrolling in calculus.

6. Discussion and conclusion

Our analysis (by means of a series of regression discontinuities front-loaded by a propensity measure) yielded no statistically significant indication that taking a college pre-calculus course helped the students' subsequent performance in college calculus. We simulated a range of scenarios for forced pre-calculus assignments, but none showed a significant benefit of the students' pre-calculus experience. If anything, there were hints that prior participation in a college pre-calculus course might even be detrimental for the calculus performance of certain groups of students. Thus, our investigation suggests a simple answer to the principal question posed in this article (Does taking a college pre-calculus course improve students' subsequent performance in college calculus?): No.

Because college pre-calculus is a substantial part of the mathematics offerings at American colleges and universities, our results have practical relevance. It is of course true that there are other legitimate goals for college pre-calculus courses, other than improving students' subsequent calculus performance.[38] As mentioned, a beneficial function of college pre-calculus may be to help keep students on mathematics or STEM trajectories who might otherwise capitulate in the face of the calculus 'hurdle'. Nonetheless, if resources are being spent on college pre-calculus courses on a grand scale, would it not be good if they were also at least somewhat effective in terms of boosting subsequent calculus performance? There appears to be much room for improvement in the pre-calculus courses.

Fife [39] notes that college pre-calculus may be ineffective owing to its close similarity to high school pre-calculus, a course that most students have already taken: students may either not take the course seriously or be discouraged by repeating it. Jarrett [8] points out that traditional approaches that did not work in high school are simply repeated anew in college pre-calculus (e.g. unrealistic word problems, excessive abstraction) – with no better results. Many students who are seeking a degree in science or engineering (the majority of those taking college calculus) want to be able to use mathematics as a tool for modelling real-world situations.[40–43] The high degree to which pre-calculus is seen as detached from the relevance of the aspects that drive them towards acquiring a STEM degree is problematic. A pertinent example was related to one of the authors by a National Science Foundation programme officer about a visit to a college pre-calculus classroom during which the tangent function was discussed as a way to measure the height of a hypothetical flagpole, 'I could forgive the instructor for not actually using the method to measure an actual flagpole, but I could not forgive that he did not even once point to the flagpole outside the classroom window'. All too often, mathematics is taught as a set of concepts developed by mathematicians pursuing goals of beauty and abstraction, when in reality many students might be more interested in solving knotty, concrete technical problems rooted in the real world.

Integrating pre-calculus with calculus into a single course has long been discussed as an alternative to stand-alone pre-calculus courses and might be a promising strategy (e.g. [44,45]), as it has been implemented at Moravian College, DePauw University, and George Washington University.[8,46] Another way to approach the issue might be letting students take Calculus I in the second, rather than in the first semester.[47] In any case, preparing students for success in college calculus deserves to become a higher priority for mathematics instructors, education researchers, and policy-makers.

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Notes

1. The definition of a 'remedial' mathematics course at the college level differs by institution. Some consider any course below calculus to be a remedial course,[6] especially those that require calculus for their science and engineering majors. Others define 'remedial' or 'precollege' as courses below pre-calculus, while they label pre-calculus as 'introductory'.
2. http://www.corestandards.org/Math.

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