

Academic Senate of the California State University

Quantitative Reasoning Task Force

Final Report, September 1, 2016

Guiding Principle: Educational policy should balance access and opportunity to achieve equity.

Upon its acceptance by the Academic Senate of the California State University in September 2016, this report and its appendices will be posted under "Student Preparedness/Success" at calstate.edu/AcadSen/Records/Reports/index.shtml.

ASCSU QUANTITATIVE REASONING TASK FORCE FINAL REPORT

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EXECUTIVE SUMMARY

In its 2015–16 term the Academic Senate of the California State University (CSU) convened a Quantitative Reasoning Task Force to review the CSU's expectations for student proficiency in quantitative reasoning upon high school and college graduation, and to recommend changes to existing policies and practices. (See [Appendix A](#), Academic Senate CSU Resolution 3230-15.)

The CSU's existing standards for statewide curricula in quantitative reasoning have been in place for many years, and this suggests they may lag behind current thinking and best practices in the field. But there is also evidence indicating that these dated policies may be acting as barriers to some students, particularly those from traditionally underserved populations and in the California Community Colleges.

The work of the Task Force was guided by the principle that *any educational policy enacted by the CSU must balance access and opportunity to achieve equity*. That is, genuine equity lies in providing students from all backgrounds with equitable prospects not only for admission and graduation (access), but also for meaningful degrees that prepare them for high-value careers after graduation (opportunity).

The Task Force included faculty and administration representing the CSU, the University of California, the California Community Colleges, the California Department of Education, employers, and the Office of the Lieutenant Governor. Its final recommendations were prepared by a subset of the Task Force holding offices in the Academic Senate CSU, and designated “drafting members.” (See the Task Force membership given in [Appendix B](#).)

Members of the Task Force conducted an extensive literature review, met with invited advisors, and participated in a national forum programmed by the U.S. Department of Education and hosted at the CSU Office of the Chancellor.

This report details the final recommendations of the Quantitative Reasoning Task Force, and they are summarized here.

Recommendation I: Formulate an updated quantitative reasoning definition based on CSU best practices and reflecting national standards.

Current policy relies on the phrase “intermediate algebra” as shorthand for full college preparation through high school, and defines baccalaureate-level quantitative reasoning as the math that builds on this level. The Task Force recommends updating this definition to include other kinds of quantitative reasoning.

Recommendation II: Revise CSU quantitative reasoning requirements and adopt equitable, feasible requirements that articulate with the other segments.

The Task Force found that CSU policies with respect to admission, transfer, and graduation are unduly constrained by treating foundational quantitative reasoning as necessary for success in all kinds of baccalaureate-level quantitative reasoning. Better policies would recognize that quantitative reasoning is valuable at both levels in ways that aren't always sequential. The Task Force proposes flexible and appropriately rigorous definitions of quantitative reasoning at the foundational and baccalaureate levels to inform separate requirements at entry and at graduation. The general expectation is that California's current State Standards in Mathematics, which follow closely the national Common Core Standards, will improve quantitative reasoning proficiency in students entering CSU, the University of California (UC) and the California Community

Colleges (CCC) system. It is the hope of the Task Force that in future most students will easily surpass the Foundational Quantitative Reasoning threshold.

Recommendation III: Ensure equitable access and opportunity to all CSU students.

The Task Force recommends policy revisions to provide equitable treatment of community college transfer and native CSU students; improve access to quantitative reasoning classes relevant to a student's major, interests and career; and raise the CSU system-wide expectation for quantitative reasoning in high school from three to four years of coursework.

In each of its recommendations, the Task Force has sought equity through a balance of access and opportunity. For example, the recommendation to raise the CSU's system-wide expectation of quantitative reasoning in high school to four years of coursework stipulates that the fourth year of instruction could reinforce practice and application of prior learning in quantitative reasoning rather than broach new topics in math. (In operational terms this means the fourth year of high school quantitative reasoning might not be in Area c of the UC a–g curriculum of college preparatory courses.)

Recommendation IV: Create a CSU “Center for Advancement of Instruction in Quantitative Reasoning”

The Task Force appreciates the rapidly changing contexts of high school instruction, best practices in postsecondary education, and the skills in quantitative reasoning that CSU students will rely on after graduation. This report supports a recent resolution of the Academic Senate of the CSU calling for creation of a dedicated Center, whose task it would be to implement these and subsequent findings and to support much-needed development of high-quality instruction and curricula in quantitative reasoning throughout the state's high school, community college and public university systems.

Although presented separately here, the four recommendations are interdependent. The policy proposals in [Recommendation III](#) depend on the definitions and distinctions of [Recommendations I](#) and [II](#). The Center for Advancement of Instruction in Quantitative Reasoning ([Recommendation IV](#)) would provide a venue for the consultation and collaboration necessary for success in [Recommendations I–III](#). Members of the Task Force expressed reservations about reducing the emphasis on algebra unless rigor could be assured in other ways. The Center, to be modeled on the CSU's successful Center for the Advancement of Reading, would provide the sustained system-level attention to pedagogy, evidence of learning at entry for both freshmen and transfer students, and support for high schools offering 12th grade courses in quantitative reasoning.

INTRODUCTION TO CSU QUANTITATIVE REASONING

Current policies.

Before admission. As part of the Early Assessment Program (EAP), California 11th grade students take the *California Assessment of Student Performance and Progress in English and Mathematics*, which provides an early indication of their readiness for college, while still allowing for time to schedule additional classes in the senior year if necessary. The Early Assessment Program (EAP) is a collaborative effort among the California State University, the California Department of Education, and the State Board of Education. Currently the program uses the *Smarter Balanced Summative Assessment* in mathematics to measure student proficiency.

Upon admission. Pursuant to [Title 5 of the California Code of Regulations](#),¹ the CSU requires that all admitted students “possess basic competence in . . . mathematical computation to a degree reasonably expected of entering college students.” Further, the CSU must promptly identify students who “cannot meet such competence” and require that they remediate any entry-level “deficiencies”. To these ends, the CSU Chancellor issued Executive Order 665 [[EO 1997](#)] to establish the Entry-Level Mathematics (ELM) examination and a committee for its maintenance. EO 665 Addendum A articulates entry-level expectations:

The ELM examination tests for entry-level mathematics skills acquired through three years of rigorous college preparatory mathematics coursework (normally Algebra I, Algebra II, and Geometry).

Addendum A also provides ELM test proxies (e.g., SAT, ACT, and Advanced Placement exam scores) for establishing basic competence. In the twenty years since the creation of the ELM test, there has been a decreased emphasis on second-year algebra and an increased focus on deeper mastery of the skills developed in Algebra I and Geometry, as evidenced in the list of topics on the ELM test published at ets.org/csu/about/elm/elm_topics. In 2002 developers revised the test to include more text-based and contextualized problems to assess quantitative reasoning in different situations and for different purposes. Of great concern to the Quantitative Reasoning Task Force is the fact that corresponding scores on the ELM test proxies (such as the SAT) were not adjusted to match the new ELM test content.

Summer before freshman year. The Early Assessment Program has been nationally recognized for raising high school students’ awareness of their readiness, and contributing to increased enrollment in 12th grade math and English. But in its first decade of implementation, rates of student readiness at college entry remained flat, as documented by the proficiency reports at calstate.edu. In response the Trustees created the Early Start Program in 2010, subsequently codified in Executive Order 1048 [[EO 2010](#)], which states:

[I]ncoming freshmen who have not demonstrated proficiency in English and/or mathematics will be required to begin remediation prior to the term for which they have been admitted, e.g., summer prior to fall.

Implementation was phased in over several years, with the final phase completed summer of 2014. As of this writing, a record share of the CSU’s incoming freshmen are placed at college level, a success that the system attributes in part to the combined benefits of the Early Assessment Program and Early Start. A March 2015 report to the Board of Trustees states:

The Early Start program has successfully enhanced pre-existing campus and system efforts to improve the number of freshmen prepared for college-level mathematics and English when they begin their first term. In summer 2010, existing CSU programs improved proficiency in both English and mathematics by one percentage point resulting in 44 percent of the 2010 freshmen class starting their first term at the CSU college-ready in English and mathematics. Comparatively, summer 2014 Early Start courses improved proficiency in both English and mathematics by five percentage points resulting in 59 percent of the

¹See law.resource.org/pub/us/ccr/gov.ca.oal.title05.html.

entering freshmen class being prepared for college-level English and mathematics [Smith and Sullivan 2015].

Prior to graduation. As part of the General Education Breadth Requirements, Title 5 specifies that all graduating CSU students must complete at least 12 semester units (or 18 quarter units) that

[...]include inquiry into the physical universe and its life forms, with some immediate participation in laboratory activity, and into mathematical concepts and quantitative reasoning and their applications [Title 5 §40405.1].

CSU Executive Order 1100 mandates that courses in subarea B4 (mathematics/quantitative reasoning) of the GE breadth curriculum

shall have an explicit intermediate algebra prerequisite, and students shall develop skills and understanding beyond the level of intermediate algebra. Students will not just practice computational skills, but will be able to explain and apply basic mathematical concepts and will be able to solve problems through quantitative reasoning.

To comply with Executive Order 1100 and to qualify for the B4 designation, a course should include an intermediate algebra prerequisite. However, a review of system-wide approved B4 courses suggests that practices supporting the CSU Area B4 graduation requirement—like the Entry-Level Math examination—have evolved away from reliance on intermediate algebra. The Task Force examined system-level data and used course titles to group courses and enrollments into four kinds of curriculum:

- Algebra Not Calculus: Courses that rely on some algebra concepts without explicitly preparing the student for eventual study of calculus. Business math is one example.
- Calculus and/or Algebra: Courses in traditional math sequences culminating in calculus or coming after calculus, and which are recommended preparation for the majority of STEM majors.
- Statistics: Courses that emphasize statistical reasoning and don't necessarily prepare students for calculus. These are prevalent in some social science majors, and in some newer cases may not carry an explicit prerequisite of intermediate algebra.
- Ideas in Quantitative Reasoning: Courses that emphasize quantitative reasoning for everyday life, and which are typically directed at non-STEM majors.

Fall 2013–2015	Number of courses	Number of enrollments
Algebra Not Calculus	17	18,963
Calculus and/or Algebra	111	143,012
Statistics	66	85,585
Ideas in Quantitative Reasoning	56	32,334

TABLE 1. Mathematics/quantitative reasoning in the CSU B4 courses (see also [Appendix C](#)).

CSU campuses had an opportunity to correct these categorizations, and around a third offered minor adjustments. [Table 1](#) displays the results and shows that from fall 2013 to fall 2015, the CSU campuses offered a total of 250 courses that satisfied the Area B4 mathematics/quantitative reasoning requirement. Of these, 122—or nearly half—have titles such as “Statistics” or “Ideas in Math”, which suggest that students will not be expected to use intermediate algebra. Approximately 42% of the students who enter the CSU as freshmen take these non-algebra-intensive courses to meet their GE requirements. (However, some CSU campuses require students taking such classes to pass an intermediate algebra test prior to enrolling, possibly to comply with the above mentioned executive orders.)

Issues of inequity.

Inequity in access for developmental math CSU first-time freshmen. The intermediate algebra threshold does not reflect current CSU practice for entering freshmen. CSU freshmen may be deemed ready for B4 courses if they get a scaled score of 50 or better on the ELM exam. As the ELM exam tests for proficiency in Algebra I and some Geometry but very little Algebra II (generally understood to be synonymous with “intermediate algebra”), students who enter the CSU as “proficient” as measured by the ELM exam are not necessarily proficient in intermediate algebra.

Those who enter the CSU as “not proficient” as measured by the ELM exam are required to complete developmental math work within their first year. This coursework may or may not be held to the intermediate algebra standard (rather than the ELM exam standard) depending on which CSU campus the student attends. This variability can result in disparities of standards as applied to “proficient at entry” students versus those deemed “not proficient at entry”.

Since EO 665 prescribes that “not proficient at entry” students must complete developmental math coursework in a timely way or risk being “stopped out” from the CSU system, this disparity raises legitimate equity concerns.

Inequity in access for transfer students. In order to gain transfer admission to the CSU, community college students must provide evidence of satisfactory completion of an approved quantitative reasoning course with an explicit intermediate algebra prerequisite. Community college students have historically been placed into or out of college-level math by a variety of placement tests (depending on the campus), whose purpose is to determine whether students are proficient in intermediate algebra. (The placement methods within the California Community College System are currently under revision and new placement tools using multiple measures are being implemented system-wide. The Task Force took the currently available details on these tools into account while making their recommendations.)

Community college students are thus held to a stricter standard of math proficiency than are entering CSU freshmen. The placement process results in up to 85% of the student population taking sequences of developmental math courses. It is well documented that such course sequences—which may span as many as 3–4 courses—result in very few students ever completing a college-level math class. In fact, students who place into the lowest level of developmental math have only a 1-in-10 chance of ever doing so. (For an account of current placement policies, see [[Burdman 2015](#)].) This raises a second equity concern.

Each year, member institutions of the California Community Colleges (CCC) system submit more than 1000 course outlines to the CSU for recognition in the GE Breadth

and in the Intersegmental General Education Transfer Curriculum (IGETC) transfer patterns. Courses proposed for quantitative reasoning must demonstrate both an explicit intermediate algebra prerequisite and evidence that the course will build on algebra proficiency. (California's articulation records are stored online in the [ASSIST](#) database and can be accessed at info.assist.org.)

A query of community college courses currently approved for transfer credit in Area B4 Quantitative Reasoning returned records for 1,616 separate courses. As it did with the B4 courses offered on CSU campuses, the Task Force grouped community college courses into four kinds of curriculum, and then invited the colleges to make any corrections. Nearly a quarter of the state's 113 community colleges replied, some with minor corrections and others to say the groupings were accurate as proposed.

The results in [Table 2](#) indicate that transferable college-level quantitative reasoning classes in the community college system are less varied than those in the CSU. Approximately a quarter of the courses offered in community colleges are in "statistics" or "ideas in quantitative reasoning", compared to around half in the CSU. Although this finding doesn't take community college enrollment into account, it suggests that community colleges apply CSU Executive Order 1100 more literally than do CSU campuses. Since most graduates of the CSU initially enroll as transfer students, and since transfer students are a vital source of diversity and access to the baccalaureate, it follows that these differences in expectations and practices undermine the principle of equitable access to the CSU.

Inequity in opportunity for developmental math students. In response to the equity challenges above, some members of the California Community Colleges and a few CSU campuses have been piloting statistics pathways for students in non-math intensive majors. Under temporary approvals from the CSU General Education Advisory Committee (GEAC), these pathways counted for lower division CSU quantitative reasoning credit. At its meeting of September 2015, the GEAC heard reports of improved passage rates for students in the statistics pathways, both in GE quantitative reasoning courses and in some cases in subsequent lower division GE coursework that relies on quantitative reasoning (see [[GEAC 2015](#)] for the meeting minutes). These pathways also significantly narrowed or closed racial equity gaps in completion of baccalaureate-level quantitative reasoning courses. Such studies suggest that a pathways approach is a potential solution to the inequities of access mentioned above.

However, the GEAC and several faculty organizations have raised concerns about the effect of such pathways on the flip side of equity: opportunity. The absence of specific algebra requirements in these pathway programs raised concerns on the part of the CSU Council of Math Chairs and the GEAC about a possible erosion of the value

	Number of courses
Algebra Not Calculus	149
Calculus and/or Algebra	999
Statistics	272
Ideas in Quantitative Reasoning	196

TABLE 2. Mathematics/quantitative reasoning in the CCC's B4 courses. (See [Appendix C](#).)

of a CSU bachelor's degree. The promising early evidence of success was considered noteworthy but on its own not definitive, and prompted the creation of this Task Force. Worries about the erosion of the degree tended to take two forms:

1. At a general level, CSU faculty expressed flexibility about moving away from the intermediate algebra threshold but wished to do so in a way that ensured that future students are prepared to apply quantitative reasoning skills as educated and responsible lifelong learners in fields such as personal finance (e.g., compound interest rates); in topics found in general education classes such as environmental science or geology; or in the science, technology, engineering, and math (STEM) courses taken by a broad range of majors.
2. A second, more specific concern was that a revised threshold could result in channeling students from underserved communities into careers that are less lucrative and less secure. This concern arises from the conscious design of statistics pathways, intended as they are for students placed into remediation who plan to major in non-algebra intensive fields. A statistics pathway is not appropriate for students in STEM or business programs since it doesn't prepare students for careers in these fields, and most Task Force members were comfortable with this level of tracking students.

However, there was pointed concern that the level of quantitative reasoning preparation in the temporarily approved statistics pathways curricula could leave students unprepared for even non-algebra-intensive careers that require some algebra proficiency. For example, nursing programs that require physics would call for more algebra than a statistics pathway would provide. The Task Force also heard concerns from experts in math education about the appropriateness of statistics pathways for elementary school teachers. Since teaching and nursing are two common careers that provide an entrée into the middle class, many Task Force members felt that these concerns should be weighed carefully against the opportunity that statistics pathways offer for access to a baccalaureate degree for students in other programs.

All agreed that if students are to make meaningful choices among math pathways, they must be properly advised regarding career exploration opportunities, and have access to curricular maps and meta-major groupings to ensure that their choices reflect their own aspirations rather than an avoidance of mathematics.

The Task Force did not reach complete agreement on the merit of arguments for and against these specific concerns. However, it did acknowledge the importance of analyzing the equity implications of its recommendations, and it supported the premise that genuine equity demands both *access* to the baccalaureate and conservation of the degree's essential value for the *opportunities* it confers to recipients.

Inequitable outcomes in CSU baccalaureate-level courses in quantitative reasoning. The CSU Office of the Chancellor provided the Task Force with detailed enrollment data from the fall 2013 term through fall 2015, including pass rates for each of the courses tabulated in [Table 1](#). Student outcomes were disaggregated by ethnic and racial groups following national practice: African-American, Latino, and American Indian students are grouped together as so-called under-represented minority (URM) populations, while all other students are grouped separately, sometimes called non-URM, as a way of identifying inequitable outcomes. The findings (see [Table 3](#)) are consistent with national research, indicating passage rates for students from under-represented minority groups lag behind those of non-URM students (the achievement gap) and that this gap is

	Pass Rates by Population		Difference
	Latinos, African-Americans, American Indians	Other	
Algebra Not Calculus	70.77%	81.27%	10.50 pts
Calculus and/or Algebra	67.21%	76.89%	9.67 pts
Statistics	75.26%	84.74%	9.48 pts
Ideas in Quantitative Reasoning	79.94%	87.13%	7.20 pts

TABLE 3. CSU student outcomes in B4 courses, F13 through F15.
(See [Appendix C.](#))

larger in algebra-intensive courses than it is in quantitative reasoning courses that are not algebra intensive.

Goal of the Quantitative Reasoning Task Force. The Task Force sought to address the inequities it identified in both access and opportunity, while creating an up-to-date, transparent set of published criteria within which all public education systems (i.e., the range of institutions spanning high schools, community colleges, the California State University and the University of California) can innovate.

To attend to equity issues related to *opportunity*, the Task Force took the view that quantitative reasoning is more than just a single course taken to satisfy a general education requirement. It is the sum total of quantitative work necessary to support a student's major, interests, career and civic responsibilities.

Out of concern for equity issues related to *access*, the Task Force was careful to propose only those standards justified by their demonstrable value for learning. We also recognized that any evolving standard must integrate well with the curricula of our sister institutions, and so borrowed liberally from the high school segment as we drafted our recommendations, using the California State Standards language. Our recommendations were also informed by innovations in quantitative reasoning education in community colleges in California and nationwide.

Crucially, the Task Force recommends that the CSU shift from defining quantitative reasoning via prerequisites to a strategy of a clearly defining quantitative reasoning goals for both entering and graduating students. Such a paradigm leaves the responsibility of demonstrating that these goals are met to the different campuses and systems in collaboration with one another. This is a new focus of shared responsibility and brings us face-to-face with a range of new concerns, detailed in the rationales and implementation notes for the recommendations below. This collaboration between the systems to define quantitative reasoning will continue to develop as the national discussion on this topic evolves.

QUANTITATIVE REASONING TASK FORCE RECOMMENDATIONS

Recommendation I: Define quantitative reasoning. The Task Force proposes this general definition for quantitative reasoning:

The ability to reason quantitatively is a stable combination of skills and practices involving:

- (i) the ability to read, comprehend, interpret, and communicate quantitative information in various contexts in a variety of formats;
- (ii) the ability to reason with and make inferences from quantitative information in order to solve problems arising in personal, civic, and professional contexts;
- (iii) the ability to use quantitative methods to assess the reasonableness of proposed solutions to quantitative problems; and
- (iv) the ability to recognize the limits of quantitative methods.

Quantitative reasoning depends on the methods of computation, logic, mathematics, and statistics.

Rationale for Recommendation I. The CSU does not currently have a definition of quantitative reasoning to guide planning and practice. This definition involves three important concepts: reasoning quantitatively, demonstrating general quantitative reasoning ability, and preparation for ongoing development of quantitative reasoning abilities. It is based on, though it differs from, those found in [MAA 1994, Dwyer et al. 2003, AACU 2013, Roohr et al. 2014].

The next section applies this definition to the different contexts in which students shall be required to demonstrate their ability to reason quantitatively.

Recommendation II: Revise quantitative reasoning requirements. Assessing the ability of students to reason quantitatively depends on their educational context. The quantitative reasoning definition proposed in [Recommendation I](#) is intended to inform revised policy that (1) evaluates the general quantitative reasoning ability of students entering and graduating from the CSU, (2) articulates well with the CSU's sister segments (California public high schools, California Community Colleges, and the University of California), and (3) specifies clearly stated and achievable procedures for evaluating and improving general quantitative reasoning ability.

Such requirements must acknowledge that the world is changing and mathematics is changing along with it. The National Academies Report *Mathematical Sciences in 2025* [NAR 2016] made it clear that mathematics is broader than arithmetic, algebra, and calculus at the service of research mathematics, engineering and science:

The ongoing trend for the mathematical sciences to play an essential role in the physical and biological sciences, engineering, medicine, economics, finance, and social science has expanded dramatically. The mathematical sciences have become integral to many emerging industries, and the increasing technological sophistication of our armed forces has made the mathematical sciences central to national defense. A striking feature of this expansion in the uses of the mathematical sciences has been a parallel expansion in the kinds of mathematical science ideas that are being used [NAR 2016].

The current debate among mathematicians and the general public is whether a common quantitative reasoning set of skills and practices exists, and if so whether algebra has any part of it. Math requirements that prescribe intermediate algebra for everyone at

the foundational level or college algebra for everyone at the college level have been described as “the single-file death march that leads towards calculus” [Holm 2015]. Nationally they are being replaced by pathways that are tailored to a student’s major or career.

At the same time, algebra has also been called a “civil right” by Robert P. Moses. Similarly, Linda Rosen, CEO of Change the Equation, has stressed the importance of algebra in the workplace [Rosen 2012]:

Corporate America understands that on-the-job-training will always be needed. Cutting-edge products and ideas inevitably require employees to learn new things. But, corporate America understandably balks at on-the-job-training that covers content that should have been learned—like algebra—before joining the workforce.

Let’s not throw the baby out with the bathwater. Instead, let’s ensure that all students master algebraic thinking and problem-solving, the essence of algebra, regardless of their eventual career goals.

These remarks speak to a more practical view of the role of algebra in a student’s development, and it supports the defense of algebra as part of a liberal arts education brought by Nicholas Warner (Professor of Physics, Mathematics and Astronomy, University of Southern California) [Warner 2012]:

One of the less obvious goals in algebra is to get people to think more abstractly. Very elementary mathematics is all about “real things” and initially employs realia to help us add, subtract and multiply. From this experience we learn the language and some of the basic rules of mathematics. We abstract and generalize the experience and learn that, when we manipulate one side of an equals sign then the equality is only true if we do the same thing to the other side. Algebra makes a major intellectual leap: It names and labels things that we do not immediately know and that sometimes lie outside our direct experience. There are certainly other studies that involve abstractions like love, empathy and ethics, but in algebra we learn to handle abstractions that are not part of visceral human experience. We learn not only to be comfortable with such external unknowns but how to master them.

Such strong and seemingly divergent views of algebra’s role in quantitative reasoning point to the urgency of the task to reconsider quantitative reasoning requirements and the role of algebra in them. They suggest moreover the need for a more subtle analysis of which quantitative skills and practices are truly necessary for a given purpose.

In making that evaluation, the Quantitative Reasoning Task Force referred back to its guiding principle: the need to balance access and opportunity to achieve equity. Each time a mandatory skill is added to the “baseline,” we risk excluding students from the academy, and yet each time one is removed, we risk limiting the value of the degree pursued. The task is to define which quantitative skills practices give enough value that they are worth the risk of limiting access, and this must be done in a dynamic and changing world.

The Quantitative Reasoning Task Force sought to establish a reasonable quantitative reasoning foundation on which additional specialized quantitative skills and practices could be built in the context of a student’s interests, major, and intended career. The Task Force started with a logistical recommendation to separate the entry and exit level of quantitative reasoning.

Recommendation IIA: Separate foundational and baccalaureate quantitative reasoning requirements. The Task Force recommends ending the use of prerequisite language to impose a de facto foundational quantitative reasoning requirement. Instead it recommends defining separate foundational and baccalaureate requirements that are reasonable and equitable.

Rationale for Recommendation IIA. The Quantitative Reasoning Task Force used the definition of quantitative reasoning in [Recommendation I](#) to guide its recommendations for quantitative reasoning policy. In doing so, the Task Force identified two weaknesses of the current CSU quantitative reasoning policies:

- (1) Current policy relies on “intermediate algebra as an explicit prerequisite” as the main identifier of a course that meets the B4 requirement. To move beyond this definition a well-articulated quantitative reasoning requirement is needed to provide a reasonable level of consistency between different CSU campuses, while maintaining principles of academic freedom.
- (2) Serious inconsistencies exist between the quantitative reasoning requirements of native CSU freshmen and those of transfer students from community colleges. The inconsistencies may disproportionately and negatively impact historically underserved populations.

This rationale describes how the Task Force’s efforts to developed a well-articulated, equitable quantitative reasoning requirement led to the proposed separation of the entry and exit requirements for quantitative reasoning.

As stated in the codified expectation section, current policy requires that any B4 (mathematics/quantitative reasoning) course transferable to the CSU or UC “have intermediate algebra as a prerequisite.” Note: for the sake of concision, we use the term “quantitative reasoning” hereafter as shorthand for “mathematics/quantitative reasoning”. In doing so, we intend no devaluation of the role of mathematics in quantitative reasoning.

This statement is natural for a quantitative reasoning course taken by a student majoring in STEM (Science, Technology, Engineering and Mathematics) for whom the calculus pathway is mandatory. However, it does not make sense for the majority of students in the CSU who are taking statistics or quantitative reasoning courses to satisfy their general education requirement in quantitative reasoning. (See [Table 1.](#)) Such courses have greatly expanded in enrollment and content over the last 20 years, and the curriculum tends to be less algebraically intensive but in many respects significantly more conceptually challenging than intermediate or college algebra.

The Task Force members acknowledge that in the same 20 years the intermediate algebra threshold has served a secondary purpose as the de facto standard of “foundational quantitative reasoning proficiency.” This standard has offered a shared base on which baccalaureate quantitative reasoning courses, as well as other general education courses, can be built. Removing that criterion or changing it may have serious consequences for students and programs. Many general education courses assume the content of intermediate algebra or the “mathematical maturity” that proficiency in intermediate algebra implies. Thus, changing the status quo must be done with care. We note, moreover, that the growth in statistics and quantitative “life skills” in general education courses appears to have been encouraged by reliance on the de facto standard because CSU faculty have felt confident that students completing a general education quantitative reasoning course will possess demonstrated proficiency not only

in the skills of that particular course but also in the more general skills of the informal foundational threshold.

It is interesting to note that in [Roohr et al. 2014] the authors' proposed framework for assessing quantitative literacy in higher education is based on math content similar to the ELM. This suggests that deepening, extending, and contextualizing these skills is at the heart of college-level quantitative reasoning. This does not presuppose that students have mastery of these skills prior to college or should be denied access to college based on this list of skills, but rather that these skills should grow and deepen over time.

The Quantitative Reasoning Task force researched national best practices, interviewed colleagues from STEM and non-STEM fields, and listened to presentations from policy makers and experts in the field, including:

- Ted Mitchell, Under Secretary, U.S. Department of Education
- Catherine Lhamon, Assistant Secretary, Office for Civil Rights, U.S. Department of Education
- Philip Daro, mathematics educator and coauthor of the national Common Core Standards for Mathematics
- Bill McCallum, University of Arizona math professor and coauthor of the national Common Core Standards for Mathematics
- Robert Green, UCLA Math professor and founding member of Transforming Post Secondary Education in Math
- Tristan Denley, Vice Chancellor for Academic Affairs, Tennessee Board of Regents
- Estela Bensimon, USC Higher Education Professor & Founder of The Center for Urban Education
- Christopher Edley, Berkeley Law professor and President of The Opportunity Institute

The Task Force concluded that because the current quantitative reasoning GE requirement defines a quantitative reasoning course as one with “intermediate algebra as an explicit prerequisite”, it involves misuse of the word “prerequisite” and a misrepresentation of current practice within the CSU, and does not even reflect current best practices for undergraduate curriculum in mathematics and quantitative reasoning.²

The Task Force believes that separating foundational and baccalaureate quantitative reasoning benchmarks will create a more constructive environment within which requirements for both levels can be discussed. This separation allowed the Task Force to develop consensus definitions of quantitative reasoning requirements that balance access and opportunity.

Recommendation IIB proposes a definition of quantitative reasoning for the baccalaureate level, while Recommendation IIC proposes a definition of the foundational quantitative reasoning the CSU would expect of all students at entry.

Recommendation IIB: Define baccalaureate quantitative reasoning. To earn a baccalaureate degree from the California State University, students shall:

²*De facto* as reflected in the various GE curricula used across the CSU system. Campus implementation of the current CSU quantitative reasoning requirement for graduation conforms to many of the suggested best practices for undergraduate students pursuing baccalaureate degrees in the U.S. As GE curricula vary across the 23 campuses within the CSU, the quantitative reasoning graduation requirements are implemented differently on different campuses.

- (i) develop and demonstrate a proficient and fluent ability to reason quantitatively in a broad spectrum of the contexts defined by California State Standards for High School;
- (ii) develop and demonstrate a general understanding of how practitioners and scholars solve problems quantitatively in a range of disciplines;
- (iii) develop and demonstrate an in-depth understanding of how practitioners and scholars solve problems quantitatively in a specialized area (e.g., the major); and
- (iv) be prepared to develop their ability to reason quantitatively after graduation in the various contexts defined by personal, civic, and professional responsibilities.

Rationale for Recommendation IIB. This definition reflects the existing good practice within the CSU in which students take quantitative reasoning B4 courses appropriate to their majors, general education interests, and careers. It also acknowledges that students develop quantitative reasoning outside of their B4 courses. Students have always reasoned quantitatively in general education classes in science, business, or technology, and are increasingly asked to do so as part of critical thinking on issues of equity, sustainability, and politics.

Recommendation IIB encourages system-wide conformity in the expected quantitative reasoning ability of students graduating from the CSU without infringing on academic freedom or being so prescriptive as to stifle the distinct campus cultures that thrive in the CSU. It is framed in the language of the California State Standards and thus articulates well with our sister segments (California high schools, California Community Colleges, and the University of California). Finally, it specifies a clearly enunciated framework within which procedures for evaluating and improving general quantitative reasoning can be assessed.

Notes on implementing Recommendation IIB. The above requirement shall be managed through the existing processes that determine whether courses meet general education requirements. The B4 courses would provide the backbone of the quantitative reasoning skills while other general education classes that require quantitative reasoning (e.g., science) would deepen and broaden the student's practice. The Task Force noted that the Western Association of Schools and Colleges (WASC) has asked for upper division critical thinking or quantitative reasoning measures and [Recommendation IIB](#) lends itself to such development.

Within the CSU, courses that deepen or broaden students' quantitative reasoning significantly beyond that of the California State Standards for high school shall be deemed college-level. For example, the typical course in statistics would be college-level whereas an intermediate algebra course would not be, since the content of intermediate algebra is completely contained within the California State Standards. Moreover, a course in statistics would qualify not only as college-level, but also as a B4 course.

In contrast, a history class may use quantitative reasoning at the college level; however, it will be unlikely to develop student proficiency to the extent that the course would meet the B4 criteria. The Task Force supports the development of a general rubric which can be adapted by CSU and community college campuses to evaluate courses against B4 criteria. The delicacy of these boundaries and the inevitable controversy they will cause emphasize the need for continued dialogue and development, ideally to include faculty, evaluators, and articulation officers with guidance from a CSU Center. (See [Recommendation IV](#).)

Recommendation IIC: Define foundational quantitative reasoning. Upon entering the California State University in pursuit of a baccalaureate degree, students will be prepared to develop their ability to reason quantitatively in the broad spectrum of courses involving quantitative reasoning offered within the CSU (including, but not limited to, B4 courses). In particular, a student who has satisfied the foundational quantitative reasoning requirement shall have:

- *Demonstrated* proficiency and fluency in the combined skills found in the California State Standards for K–8, Algebra 1, and Integrated Math 1;
- *Practiced* the skills in the K-12 California State Standards for Mathematics in a variety of contexts that broaden, deepen or extend K-8, Algebra 1 and Integrated Math 1 skills;³
- *Developed* the eight Common Core mathematical practices, which are the abilities to:
 - Make sense of problems and persevere in solving them
 - Reason abstractly and quantitatively
 - Construct viable arguments and critique the reasoning of others
 - Model with mathematics
 - Use appropriate tools strategically
 - Attend to precision
 - Look for and make use of structure
 - Look for and express regularity in repeated reasoning.

Rationale for Recommendation IIC. While the Quantitative Reasoning Task Force found consensus fairly easily around the definition of the baccalaureate quantitative reasoning requirement, the boundaries of the foundational quantitative reasoning requirement were more problematic, as their identification required looking at what quantitative reasoning preparation a student would need in a broad range of majors, general education interests, and careers, as well as in civic life.

Moreover, this definition relates the CSU to all segments of California's public education system, as illustrated in a number of possible scenarios:

- James is a high school junior whose test results indicate he is only "conditionally proficient" in foundational quantitative reasoning. To satisfy the condition for full readiness, he would benefit from senior year course options to reach full proficiency for quantitative reasoning in the CSU.
- Samantha is a community college student hoping for an Associate Degree in Psychology. She did not graduate from high school. She needs a well-designed pathway or series of courses to achieve foundational and baccalaureate proficiency before transferring to the CSU. As much as possible this coursework should relate to her major and interests.
- Maura is a CSU entering biology major who is not proficient in foundational quantitative reasoning. She needs some developmental math coursework to prepare her for pre-calculus.
- José is an entering sociology major who is not proficient in foundational quantitative reasoning. He needs some developmental math coursework to prepare him for statistics.

³Including quantitative reasoning skills as practiced in high school curricula outside of mathematics.

The foundational quantitative reasoning requirement needs to address this full spectrum of students and to support a broad range of non-algebra intensive majors, general education interests, and careers, while preparing students for civic life.

In trying to identify the correct threshold for the foundational quantitative reasoning requirement, the Quantitative Reasoning Task Force relied on multiple sources, including the report [ICAS 2013] of California's Intersegmental Committee of Academic Senates (ICAS), the California State University Council of Math Chairs' *Statement on Entry Level Mathematics and Statway* [CSUCMC 2015], and evaluations of the California State Standards.

Initially the Task Force found the language of "mastered" and "practiced" (commonly used in secondary math standards) was helpful in defining the foundational quantitative reasoning threshold. It allowed the group to focus on what skills and practices were foundational and subsequently to discuss the necessary depth and breadth of student learning. In these discussions the group used "mastered" to describe internalized learning that students are prepared to apply confidently in a range of settings. The Task Force does not intend to recommend individual test instruments or any threshold scores (e.g., 80% or 90%) that may be implied by the word "mastery" in other sectors of education. For this reason "mastered" was replaced by "proficient and fluent" in item (i) of [Recommendation IIB](#).

To get a broad and national view, Task Force members looked at reports from professional mathematics and statistics organizations, national studies, and leaders in STEM and non-STEM professions. (See [Appendix D](#) for a full bibliography.) The Quantitative Reasoning Task Force paid particular attention to majors that lead to careers in nursing, teaching, law enforcement, and business, as these non-STEM careers typically attract students who hope to move into the middle class. It also compared the quantitative skills students would need for such majors to the California State Standards for mathematical skills and practice.

The Standards of Mathematical Practice, spelled out in the California State Standards, provide a broad framework of habits of mind that, when practiced in contexts requiring mathematical skills, are *quantitative reasoning*. The mathematical skills set forth in these Standards grow upon one another in the K-12 curriculum, forming a tall, narrow tree of knowledge. In fact, this construct is central to the national Common Core Standards (on which California's are based), where skills are developed through just a few "progressions": number systems, expressions and equations, functions, geometry, and statistics and probability.

In general, the Common Core's progressions resist the idea of mathematics as a list of topics because lists quickly become too long for students to keep in their active memories. Rather the progressions invite students to recognize underlying principles. This recognition "shrinks" the mental real estate required for memorization while deepening mathematical understanding [Stevenson 2015].

Because the mathematical knowledge tree is narrow, defining foundational quantitative reasoning means deciding which branches of the curriculum are fundamental to our purpose of buttressing student opportunity while maintaining maximal access to higher education.

The Task Force looked for a foundational quantitative reasoning threshold that would guarantee the mathematical skills necessary for non-algebra intensive majors, quantitative reasoning skills for life (typically taught in an "ideas in math" class), and a very

narrow list of skills and knowledge that members considered necessary for a liberal arts education.

Statistics is a non-algebra-intensive baccalaureate quantitative reasoning course. Recent work suggests that in the context of the California State Standards, to be successful in Statistics a student would need to be proficient in most of the K–8 curriculum as well as in several topics from the Algebra 1 or Integrated Math 1 curriculum. For example, a student needs to be able to evaluate algebraic expressions in order to calculate numerical summary statistics, test statistics, confidence intervals, z-scores and regression coefficients in statistics [Peck et al. 2015].

Additionally, CSU graduates in any major will likely need to manage a business budget or choose among mortgage options. Thus, they should have the necessary skills to be ready to learn about personal and business financial models: simple and compound interest, as well as the fundamentals of cost, revenue, and profit. This future learning might happen in a quantitative reasoning class, a GE elective on sustainability, or even on the student's own after graduation, but the foundations are necessary. Readiness to learn financial models requires the skills found in Algebra 1 or Integrated Math 1, such as the ability to “interpret functions that arise in applications in terms of the context” or “construct and compare linear and exponential models and solve problems”.

In the course of its analysis, the Quantitative Reasoning Task Force found that the correct foundational quantitative reasoning requirement for mastered skills lies quite close to the combination of the K–8 plus the Algebra/Math 1 curriculum. This standard concurs with those of Georgia, Texas, Indiana, and Maryland and is close to the Entry-Level Mathematics threshold supported by the CSU Council of Math Chairs. In particular, the ELM threshold does not require exponential models at all, but it does require students to manipulate expressions involving ratios. The Quantitative Reasoning Task Force feels that such distinctions can be readily reconciled via broad consultation over the 2016–2017 academic year. In many cases, it may be a matter of defining more specifically what level and depth is intended by the standards.

The Task Force strongly recommends that the CSU operationalize this definition of foundational quantitative reasoning by drawing wherever possible from the California State Standards.

At the same time, the Task Force advises the CSU to monitor the impacts of this recommendation on student attainment and equity, and to continuously evaluate the connections between skill requirements and their rationales. For example, it is reasonable to say that students should be able to “evaluate algebraic expressions,” “compute compound interest,” or “be able to solve a linear equation in one variable” in a simple interest formula. However, it was the consensus of the Task Force that it would be unreasonable to require a student in a non-algebra-intensive field to solve for time in a compound interest formula, $A = P(1 + r/m)^{mt}$, by using logarithms. The Task Force acknowledges that the proposed recommendation is just one iteration in a series of refinements and alterations.

Implementation notes for Recommendation IIC. Just as with the current policies related to the ELM test, a standard for foundational quantitative reasoning is not intended as a CSU admissions requirement for first-time freshmen. Rather it is an expectation for entering students, which if not met at entry must be satisfied through developmental math coursework under existing guidelines.

Any measure of foundational quantitative reasoning proficiency should include as part of its criteria a proctored assessment of the skills in question.

In the short term, the foundational quantitative reasoning requirement could be implemented using the existing Smarter Balanced/SAT/ACT/ELM structure, although the thresholds of the SAT and ACT should be revised, since they are based on the old intermediate algebra standards. The Quantitative Reasoning Task Force recommends that an implementation team review this foundational quantitative reasoning recommendation in fall 2016, with particular attention to feasibility, relevance, and equity. The team should recommend any necessary changes to the Smarter Balanced/SAT/ACT thresholds and possibly to the ELM content as determined by the CSU.

The Quantitative Reasoning Task Force recognizes that quantitative reasoning as applied to a consideration of majors, careers, and civic life is an evolving construct, and that its meaning in the context of foundational and baccalaureate requirements will need to be revisited regularly. The Task Force calls on the CSU to develop a streamlined process for periodic refinement of these requirements, using evidence-based methods that take into account national trends in addition to the realities of the California public education system.

To that end, the Task Force calls upon the professional societies from both STEM and non-STEM fields to work with the Transforming Post Secondary Education in Mathematics organization (TPSE Math) to conduct an in-depth study of the logical progression in math pedagogy between the skills of Common Core Math and those of baccalaureate quantitative reasoning. Such a study has already been done [Peck et al. 2015] in the context of statistics classes for sociology and psychology, and it should also be done for “quantitative reasoning” classes and for meta-majors (see [Lumina 2014]), more broadly. Doing this in piecemeal fashion, campus by campus will merely produce inconsistent results or replicate work that should be shared. Instead, such an in-depth study is an endeavor that should engage a broad range of national experts and practitioners. Once the work is done broadly, individual departments, campuses and systems can tailor the results to their own environments based on their students, resources, and academic goals. In particular, such work could be used at the time of the next review of the foundational quantitative reasoning requirement.

Recommendation III: Ensure equitable access and opportunity to all CSU students.

Recommendation IIIA: Promote equity, access and opportunity. The Task Force recommends that equitable policies be established to provide transfer and developmental math students with increased access to quantitative reasoning courses that can open up opportunities in these students’ majors, interests, careers, and civic lives.

Rationale for Recommendation IIIA. This recommendation addresses the circumstances described in “Issues of inequity” by calling on the CSU to change its policies so that transfer students and CSU first-time freshmen requiring developmental math coursework are held to the same foundational and baccalaureate quantitative reasoning proficiency standards.

Along with these changes, the Task Force encourages the CSU to ensure that

- all CSU campuses provide students with at least one B4 course that has no prerequisites beyond the foundational quantitative reasoning requirement, and that such courses be relevant to a broad range of majors and interests (e.g. statistics, ideas in quantitative reasoning, or mathematics for life);
- students with algebra intensive majors, interests, and career goals be required to take additional mathematics at either the baccalaureate or developmental

level prior to taking the appropriate B4 course as necessary. (For example, a student may need intermediate algebra or college algebra prior to taking pre-calculus or mathematical methods in business.)

Implementation notes for Recommendation IIIA. The CSU needs to develop rubrics or other means to determine whether successful completion of a course, pathway, or sequence of courses should be sufficient to demonstrate foundational quantitative reasoning proficiency.

The implementation of Recommendation IIIA will also require consideration of how students may experience these policy changes in the different contexts of high school, community college and university. In the case of high school, we make the following, additional recommendation in support of a recent resolution on the part of the Academic Senate CSU (ASCSU).

Recommendation IIIB: Require four years of high school quantitative reasoning. The Quantitative Reasoning Task Force recommends that four years of high school quantitative reasoning coursework be required as part of the CSU admissions criteria (per ASCSU Resolution AS-3244-16/APEP).

Rationale for Recommendation IIIB. As the ASCSU noted in the rationale for Resolution AS-3244-16/APEP, the success of incoming students is maximized when students maintain their exposure to mathematics/quantitative reasoning. As is the case with a second language, mathematical skills decline from lack of use, and it is important that students continue practicing and developing quantitative abilities throughout their academic careers. In a number of settings, including the CSU Admission Handbook and through [CSU Mentor](#), the CSU already recommends four years of mathematics,⁴ even though only three years are required. The standing ICAS recommendation in the “*Statement on competencies in mathematics expected of entering college students*” similarly states [[ICAS 2013](#)]:

For proper preparation for baccalaureate level coursework, all students should be enrolled in a mathematics course in every semester of high school. It is particularly important that students take mathematics courses in their senior year of high school, even if they have completed three years of college preparatory mathematics by the end of their junior year. Experience has shown that students who take a hiatus from the study of mathematics in high school are very often unprepared for courses of a quantitative nature in college and are unable to continue in these courses without remediation in mathematics.

It is important to note that the fourth-year mathematics course called for by the CSU resolution would not necessarily be a fourth course in Area c; it must be a–g compliant, but it could be a course approved in Area g.

Other states in the U.S. already require a fourth year of mathematics for admission to their state university systems. For example, effective with the class entering in the fall of 2015, students in Maryland are required not only to complete four years of mathematics for entry to any of the state’s public universities, but those who complete Algebra II prior to their final year must complete the four-year mathematics requirement

⁴See [csumentor.edu/planning/high_school/subjects.asp](https://www.csumentor.edu/planning/high_school/subjects.asp).

by taking a course or courses that utilize non-trivial algebra [St. George 2014].⁵ The Maryland policy was based in part on the report “*Coming to our senses: Education and the American future*” [Kirwan et al. 2008], which found that the academic intensity of the high school curriculum was the most important predictor of college success, and so recommended four years of college preparatory mathematics.

These findings and prescriptions are not new. Kirst argued in “*Overcoming the high school senior slump: New education policies*” that high schools should redesign their senior year courses to serve as gateways to general education requirements students would likely encounter in their first year of college and emphasize the importance of taking senior-year math courses [Kirst 2001]. He also recommended that colleges should include a senior-year math course in their admissions requirements.

There is a strong correlation between taking more mathematics in high school and being college-ready upon arrival at the university. Studies have documented that

1. SAT-Math and ACT-Math scores improve as the number of years of high school mathematics increases (see [SAT 2013]–[SAT 2015]);
2. the likelihood of needing remediation decreases and the likelihood of completing general education quantitative reasoning requirements increases as students take more high school mathematics (see, e.g., [USHE 2015]).

Finally, many former high school students, with the clarity of 20/20 hindsight, recognize that they should have taken more (or more difficult) mathematics courses in high school. A “one year later” survey of 1,507 high school graduates found that 44% of those students wish they had taken different courses in high school. The most frequently expressed regret (40% of this group, or more than one in every six students surveyed) was that they hadn’t taken more or higher-level mathematics courses [Hart 2011]. (For further background on the subject of mathematics courses in the senior year of high school, see Appendix E.)

Implementation notes for Recommendation IIIB. If the CSU adopts this admission requirement, there will be a natural implementation phase of at least three to four years. The CSU cannot impose this requirement on students already enrolled in high school; it will be operational only as the next 8th grade class enters the 9th grade. With this in mind, the CSU needs to move forward by communicating its intention to all stakeholders and interested parties as soon as possible.

The CSU will be in a better position to assist high schools in meeting the new requirement with existing Area c and other appropriate courses as well opportunities for professional development if the system supports creation of a Center for the Advancement of Instruction in Quantitative Reasoning. The Center would be charged with developing a modular course patterned after the Expository Reading and Writing Course, which was designed to reduce remediation needs in English.

More than 60 percent of students advancing to the CSU from high school already complete four years of math. Moreover, many California high schools already offer such a 12th grade course in quantitative reasoning. The goal is to fill in the gap and overcome what might otherwise be a one- or two-year hiatus in students’ use of acquired quantitative skills.

⁵For admissions requirements to the University System of Maryland, see: usmd.edu/newsroom/news/1021; admissions.umd.edu/requirements/Freshmen.php; undergraduate.umbc.edu/apply/freshmen.php.

How students satisfy the requirement for 12th grade quantitative reasoning would depend on individual proficiency upon entering the senior year. It could be an a–g course that introduces new material, or a course that reinforces learning from earlier years.

High school quantitative reasoning course definition. If the a–g required coursework in math is being completed in the senior year with a course such as Algebra II or Integrated Math III, then this course will count as the student’s fourth year of quantitative reasoning. If the a–g required coursework in math is being completed in the junior year, then the student must complete math-based quantitative coursework in the senior year. This requirement may be met in one of several ways:

- by completing an advanced level math course (pre-calculus, math analysis, calculus);
- by completing an Area c or g course in statistics, quantitative reasoning, mathematics or computer science or any other approved math-based quantitative Area c or g course; or
- by completing an algebra-based Area d science course (e.g., chemistry or physics).

In California, the State Standards determine what students in grades K–12 should know and be able to do in mathematics, and the Smarter Balanced Assessment is used to assess attainment of the standards. Any CSU-admissible student must have completed the full California State Standards for K–12, and so will have fulfilled the parts of the foundational quantitative reasoning requirement that oblige students to have “practiced the skills in the K–12 California State Standards” and to have “developed the eight Common Core mathematical practices”.

What remains is to determine whether a student has “demonstrated proficiency and fluency in the combined skills found in the California State Standard curriculum for K–8, Algebra 1, and Integrated Math 1.” As stated earlier, Title 5 requires that the CSU identify “as quickly as possible” those admitted students “who cannot demonstrate . . . such basic competence” and require them to engage in what is commonly called remediation.

The junior year Early Assessment Program and Smarter Balanced Assessment results are the means for informing CSU-bound students of their quantitative reasoning status “as quickly as possible” (Title 5). The CSU designates entering students as proficient, conditionally proficient, or not proficient in quantitative reasoning for purposes of preparation for the CSU baccalaureate. By learning their proficiency status a year before they graduate from high school, CSU-bound students can proactively use their senior year to engage in quantitative reasoning coursework to help them attain proficiency prior to admission.

Below are three statements of proficiency designations and recommendations. (Note that we use the term “CSU math-eligible” to mean that a student has not only met the mathematics admission requirements to the CSU but is also ready for college-level work.)

For purposes of the recommendations below, the assumption is that [Recommendation IIIA](#) will be implemented. That is, in their senior year, students should enroll in a quantitative reasoning course as determined by their junior year Smarter Balanced Assessment proficiency status in order to reduce or eliminate the need for developmental math coursework in the CSU and at participating California Community Colleges.

- Foundational quantitative reasoning *proficient* students:
 - These students shall take any high school quantitative reasoning class as a senior.
 - They will be CSU math-eligible and will not require developmental math at the CSU or at any of the participating California Community Colleges.
- Foundational quantitative reasoning *conditionally proficient* students:
 - These students shall take an Area c or an appropriate high school quantitative reasoning course.⁶ Alternatively, such students may take any quantitative reasoning high school course in conjunction with a CSU-approved method for determining foundational quantitative reasoning proficiency.
 - Students who pass the Area c high school quantitative reasoning course or an approved equivalent high school course shall not be required to enroll in developmental math at the CSU or at any participating California Community Colleges.
- Foundational quantitative reasoning *not proficient* students:
 - These students shall take any high school quantitative reasoning course (however, Area c or g is recommended) in conjunction with a CSU-approved method for determining foundational quantitative reasoning proficiency.
 - Students deemed foundational quantitative reasoning proficient via any CSU-approved method shall not be required to enroll in developmental math at the CSU or at any participating California Community Colleges.

As discussed above, the implementation of fourth-year math classes and the attendant proficiency protocol is an ambitious endeavor—one that will take time, collaboration, resources, and most importantly an attention to equity. The Task Force recommends that the time frame to implement this requirement be extended far enough to allow high schools the time needed to develop capacity. It further recommends that the CSU and CCC partner with high schools and create a Center charged with developing appropriate curricula, assessing the outcomes of that curricula, and using the evidence to inform revisions of the curricula.

Recommendation III C: Ensure early and appropriate quantitative reasoning courses for CSU first-time freshmen. The Task Force recommends reevaluating quantitative reasoning requirements in the context of the student's educational goals and proficiency at entry. For first-time freshmen in the CSU, it therefore recommends:

- *Foundational quantitative reasoning proficient* students shall take a baccalaureate quantitative reasoning class within the first two terms at the CSU. Options shall exist in the context of the student's major and interests.
- *Foundational quantitative reasoning not proficient* students shall demonstrate proficiency within two terms of enrollment via a CSU-approved method. They shall take a baccalaureate quantitative reasoning class within two semesters of demonstrating proficiency. Options shall exist in the context of the student's major and interests. This recommendation is intended to accommodate co-requisite remediation, at the option of the institution providing the instruction.

⁶This represents an expansion of the options for students to fulfill the conditional exemption with appropriate high school courses instead of only Area c courses. An AP computer science course could qualify in this category. Courses without Area c status would have to go through existing CSU and UC approval processes.

Rationale for Recommendation IIIC. As pointed out in [Recommendation IIIB](#), students in algebra intensive fields like STEM or business may be required to take additional mathematics at either the college or developmental math level. This presents an interesting challenge for developmental math grades, as illustrated in the following scenarios:

- Maura is a CSU entering biology major who is not proficient in foundational quantitative reasoning. In fact she requires two semesters of developmental math work.
 - In her summer Early Start math class she is not able to apply herself fully because she is working 40 hours per week as a pharmacy checkout clerk. She makes sufficient progress to fulfill the Early Start requirement but does not improve her fall math placement.
 - In fall, she receives credit in Developmental Math 1 For Algebra-Intensive Majors. (This is a new category of developmental math course, proposed as part of [Recommendation IIIC](#). Maura would be enrolled in it because biology is considered an algebra-intensive major.)
 - In spring, she makes progress but not enough to earn credit in Intermediate Algebra. However, her average over the course of the semester does indicate that she is proficient in foundational quantitative reasoning.
- James is a CSU entering sociology major who is not proficient in foundational quantitative reasoning. In fact, he too requires two semesters of developmental math work.
 - In his summer Early Start math class, he is not able to apply himself fully because he working 40 hours per week as a receptionist in a health clinic. He makes sufficient progress to fulfill the Early Start requirement but does not improve his fall math placement.
 - In fall, he receives credit in Developmental Math 1 For Non-Algebra-Intensive Majors.
 - In spring, he earns credit in Developmental Math 2 For Non-Algebra-Intensive Majors, a class that teaches no more content than is necessary for proficiency in foundational quantitative reasoning.

James and Maura may be comparable in their foundational quantitative reasoning abilities. Neither one should be stopped out. However, a grade of “credit” in Maura’s spring intermediate algebra class would falsely depict her as ready for pre-calculus or college algebra. For such a student, an alternative to the traditional “credit” versus “no credit” grade is surely preferable. One model might be to use the grade “P” to denote that a student has demonstrated proficiency in foundational quantitative reasoning. Such a grade would leave Maura, the biology major, with a choice: either switch to a major requiring a non-algebra intensive coursework, or remain a biology major and repeat Intermediate Algebra.

Recommendation IIID: Establish equitable articulation of quantitative reasoning credit for transfer students. Community college students should be assessed by the community colleges as proficient or not proficient in foundational quantitative reasoning in alignment with the standards above. Prior to transfer, they should demonstrate foundational quantitative reasoning proficiency and earn the appropriate minimum grade in a course that transfers for B4 credit.

Such students will not necessarily be considered proficient in baccalaureate quantitative reasoning, as certain campuses may require upper division work for this designation.

Articulation for foundational quantitative reasoning proficiency will follow the existing approval process for B4 transfer approval. The Task Force supports the creation of options for both foundational and baccalaureate quantitative reasoning that teach skills and practices in the context of the student's major and interests.

Implementation notes for Recommendation IIID. To provide more equitable access to the CSU and to ensure that students are ready for the rigors of baccalaureate work, the Task Force has replaced intermediate algebra requirements with a foundational quantitative reasoning requirement. To meet the needs of all community college students who plan to transfer to the CSU, these new standards may require new approaches.

Students who are not deemed proficient in foundational quantitative reasoning by the community college assessment process will need opportunities to obtain these skills prior to transferring to the CSU. These opportunities may be embedded in, or taught as a co-requisite for, a B4 transfer level quantitative reasoning course, or they may be achieved in separate coursework. Coursework designed to address the foundational quantitative reasoning requirement should provide opportunities for students to deepen and broaden quantitative reasoning skills in a wide variety of contexts from the K–12 curriculum, as well as frequent opportunities to engage in learning experiences that promote the Common Core's mathematical practices.

The Task Force supports initiatives to ensure more equitable ways to bring post-secondary education to California's students by creating new quantitative reasoning pathways (such as those developed by the Carnegie Foundation for the Advancement of Teaching and the California Acceleration Project). The revised quantitative reasoning requirements, which bring the official position of the CSU much closer to the curricula developed in many pathways, are intended to give guidance for developing such innovations and therefore to eliminate the need for further exceptions and waivers.

Recommendation IV: Create a CSU "Center for the advancement of instruction in quantitative reasoning". As soon as possible, the CSU should create a Center for Advancement of Instruction in Quantitative Reasoning to act on the Task Force's current and subsequent findings, and to support the high-quality instruction in high schools, community colleges, and public universities that will better serve the state.

Rationale for Recommendation IV. The Task Force appreciates the rapidly changing contexts of high school instruction, best practices in postsecondary education, and the skills in quantitative reasoning that CSU students will rely on after graduation. There is a need for ongoing, consistent and coherent oversight of statewide efforts to make progress in mathematics education.

Recommendations IIIA–IIID propose profound changes to policy whose implementation will depend on deeper and more sustained partnerships with CSU colleagues in California's public schools, community colleges, and the University of California. We remark that the all-purpose label "intermediate algebra" has almost certainly conveyed a false sense of sequential learning in quantitative reasoning, while exacerbating disturbing inequities across the state. But historically it had the virtue of being unambiguous. Moreover, once faculty had set the ground rules, day-to-day operation could potentially be relegated to other segments.

By contrast, a more equitable, sophisticated and responsive expectation for quantitative reasoning at entry and graduation will be harder to "outsource". In fact, the CSU will need to take action to reconsider the notion of "intermediate algebra" and replace it with meaningful determinations of readiness at entry and transfer.

The Task Force believes that its recommendations are an important step toward such committed participation. The CSU has the capacity to bring to scale a more defensible set of benchmarks for student attainment that are informed by the California State Standards, bolstered by a universal expectation for quantitative reasoning in the 12th grade, and developed at the baccalaureate level in ways that are fair for CSU and community college students of all backgrounds.

The Center could also be an important source of intersegmental professional development and research into student flow across California's educational sectors, giving faculty the means to monitor and adjust the definitions of foundational and baccalaureate quantitative reasoning proposed here.

Implementation notes for Recommendation IV. The model for the proposed Center is the CSU Center for the Advancement of Reading, which for ten years has led development and deployment of a 12th grade Expository Reading and Writing and Course (ERWC) across the state. The ERWC has been nationally recognized for its success in improving college readiness in English, a track record that most observers ascribe to three factors in particular:

1. stable, central administration of courses that nonetheless benefit from local innovation and customization;
2. continuous development and refinement of curriculum, not just at the 12th grade level but also leading up to it, with scaffolded modules that begin as early as middle school;
3. built-in professional development for high school teachers.

The CSU Center for Advancement of Instruction in Quantitative Reasoning would be designed along similar principles, with the belief that student proficiency will be improved not by more exposure to advanced or esoteric topics in math, but by deeper and more varied practice in the concepts already learned.

The Center for the Advancement of Instruction in Quantitative Reasoning would also encompass an additional mandate: to add critical oversight and guidance for CSU and community college educators seeking to teach quantitative reasoning at the baccalaureate level. The Task Force believes the CSU's own Colleges of Education and Math Council could provide the necessary follow-through for this work as they educate the next generation of math teachers.

Over the course of its literature review and in conversations with every one of its advisors, the Task Force repeatedly encountered this message: CSU students don't need more math at entry, nor should they necessarily be expected to fulfill more requirements for many of the CSU majors. Instead, students need more proficiency in the math they already have. Requiring a fourth year of quantitative reasoning in high school and calling on our colleges and universities to broaden their conception of quantitative reasoning are important steps in the right direction. These strategies would be greatly enhanced, moreover, by the founding of a Center whose specific focus would be depth and mastery in learning.

TOPICS FOR FURTHER STUDY

A. The Task Force urges the CSU to conduct further studies on the use of "multiple measures" of college readiness in quantitative reasoning (for example, using proficiency as measured by high school grades in addition to single-administration test measures such as the SAT or ACT). It also wishes to call attention to a significant finding: by

treating all quantitative reasoning as sequential and relying on standardized testing as the main measure of readiness, current policy may have disparate impacts on students from diverse backgrounds or on those who begin at community colleges. In particular, an updated reliability and efficacy study should be done on the ELM test. Also, data should be analyzed to determine correct SAT and ACT threshold scores for foundational quantitative reasoning proficiency.

B. Soon after its formation the Center should bring together (1) faculty in math and other quantitative disciplines and (2) representative staff in admissions, testing, evaluation, and articulation, and (3) educators at the high school level, who can develop rubrics for the determination of proficiency at entry and transfer.

C. The Center should lead development of a quantitative reasoning course in the 12th grade analogous to the Expository Reading and Writing Course for high school seniors in Area c or g (calstate.edu/eap/englishcourse). The development should be informed by the numerous, very encouraging local examples of such courses in high school and postsecondary partnerships around the state.

The new, state-level course should be made available to high school teachers in modules that apply the skills to be mastered in Algebra/Math I and others that are introduced in the full California State Standards K–12 curriculum. Importantly, the course should have a strong focus on preparing students to engage in quantitative reasoning across a wide range of majors, interests, and careers, including, but not limited to teaching, nursing, law enforcement, information technology, sustainability, liberal studies, and social sciences.

Two prominent features of the ERWC project were robust CSU faculty involvement in course development and high-value professional development for faculty and high school teachers involved in the project's implementation. We call for the same in any forthcoming Quantitative Reasoning high school model and roll-out. We also recommend that the CSU establish a permanent position and Quantitative Reasoning Board to oversee quantitative reasoning improvements as well as issues of articulation and professional development across the CSU system.

Given the recent ASCSU resolution (May 2016) calling for the establishment of a center for mathematics instruction, such a center may be the appropriate home for development and oversight of the project. (See [Appendix F](#).)

D. Development and implementation of an upper division critical thinking assessment process that combines quantitative and expositional reasoning.

Appendices

APPENDIX A: ACADEMIC SENATE CSU RESOLUTION 3230-15

Establishing a Task Force on the Requirements of CSU General Education (GE) Mathematics/Quantitative Reasoning (B4) Credit

Resolved: That the Academic Senate of the California State University (ASCSU) appoint a task force to address two fundamental questions.

- (a) Can the pre-requisite content for the CSU GE B4 course be met concurrently with achieving the CSU GE B4 standards?
- (b) What should be the pre- (potentially co-)requisite content for quantitative reasoning and mathematical competency (CSU GE B4)?⁷

And be it further

Resolved: That the ASCSU define the membership of this task force to potentially include:

- (a) a member of the General Education Advisory Committee (GEAC) Statway advisory group;
- (b) another member of the GEAC;
- (c) a member of the Academic Affairs (AA) Committee;
- (d) a member of the Academic Preparation & Education Programs (APEP) Committee;
- (e) a representative of the Math Council;
- (f) a faculty member who teaches B4 outside of mathematics;
- (g) a California Acceleration Project (CAP) or Statway instructor;
- (h) a member of the Entry Level Mathematics (ELM) test development committee;
- (i) a representative of the CSU Office of the Chancellor;
- (j) a representative of the Academic Senate of the California Community Colleges (ASCCC);
- (k) any other interested ASCSU faculty member.

Resolved: That the ASCSU distribute this resolution to the University of California (UC) Board of Admissions and Relations with Schools (BOARS) leadership, the General Education Advisory Committee (GEAC), the CSU Math Council, the Academic Senate of the California Community Colleges (ASCCC) Leadership, and Executive Vice Chancellor Loren Blanchard.

Rationale: Five years ago the Chancellor's Office General Education Advisory Committee (GEAC) approved a limited pilot program within the California Community Colleges in order to assess the viability of meeting CSU GE B4 quantitative reasoning requirements with a two-course integrated statistics sequence. This sequence bypasses the existing intermediate algebra proficiency in quantitative reasoning required by Executive Order (EO) 1100 as a prerequisite to CSU GE B4 courses. At its September 2015 meeting GEAC agreed to extend the pilot (at seven CCC districts) for an additional three years and invited other CCC districts to submit proposals utilizing curricular innovations in statistical pathways. In addition, GEAC called for the establishment of

⁷Executive Order 1100 specifies Intermediate Algebra; the math council statement advocates for ELM content; Statway includes a lesser amount of algebra.

a task force to include disciplinary experts to review existing B4 standards in light of the fact that some of these statistics based-pathways did not include a requirement to demonstrate proficiency in intermediate algebra prior to the award of B4 GE credit.

General education curricular standards are the province of the faculty and an expansion of the pilot has implications for CSU admissions and graduation standards and thus will rely on ASCSU action. The potential expansion of the GEAC pilot project on integrated statistical pathways for underprepared students generates a need to view the potential consequences of systemic changes to admissions standards and to EO 1100. Any potential changes will influence the minimum requirements for granting of a degree from the CSU.

Reducing achievement gaps and improving student success in meeting pre-baccalaureate and CSU GE mathematics/quantitative reasoning (B4) requirements are currently problematic. The traditional developmental pathway often constitutes a “leaky pipeline” in terms of success. As a result many students will never qualify for transfer because they cannot complete the prerequisites to CSU GE B4 requirements. Integrated statistical pathway programs such as the Statway pilot and the California Acceleration Project were established to increase the number of community college students who would satisfy the CSU GE B4 requirement. There exists early work that illustrates the effectiveness of integrated statistical pathways (e.g., Carnegie Statway, California Acceleration Project, etc.) in reducing achievement gaps and improving student success as measured by pass rates. These efforts, however, do not achieve the levels of proficiency in intermediate algebra that are currently required for CSU freshman admission and thus introduce the specter of a “lesser degree” via lowering of academic standards.

The CSU Math Council, in their statement of April 2015, advocates that all students, at a minimum, attain knowledge of content as defined by the ELM requirements prior to the award of CSU GE mathematics/quantitative reasoning (B4) requirements. The statement reads in part:

We oppose the replacement of elementary or introductory statistics courses at CSU campuses by any program or pathway course lacking an explicit prerequisite or co-requisite that subsumes the content of ELM. Such pathway courses include Statway. While the statistics content of Statway is totally aligned with the standard curriculum in elementary statistics, the pre-college mathematical content of Statway by itself does not meet the ELM standards and does not prepare students for college level courses. Hence Statway in its present form does not satisfactorily accomplish remediation and GE QR [quantitative reasoning/B4] in a single track, thereby pointing to the need of having all ELM content in a prerequisite or co-requisite.

There are unresolved discrepancies among the prerequisite B4 requirement (currently “Intermediate Algebra,” per EO 1100); the potential use of ELM content (per the Math Council Statement); and the absence of any such pre/co-requisites for the CSU-approved Statway pilot project (and potentially other CSU-approved projects). This resolution attempts to address these concerns.

On the question of whether or not the pre-requisite knowledge could be achieved concurrently with the other B4 requirements, the answer is likely “yes” given the existence of “stretch” courses in which the content of a single course is stretched over multiple terms to allow inclusion of pre-baccalaureate material. It remains an open

question whether or not the current pre-requisite (possible co-requisite) content should be Intermediate Algebra (per EO 1100), the material covered by the ELM exam (per the Math Council statement), or another standard (per “just in time” delivery of algebra via Statway).

A related issue of whether CSU GE B4 standards themselves could be satisfied by meeting one of two pathways (possibly STEM vs. non-STEM, quantitative-based vs. statistically-based, etc.) should also be addressed once the issues touched on by this task force have been resolved.

Useful definitions and contextualization. Title 5 requires “inquiry into mathematical concepts and quantitative reasoning and their applications” (CCR §40405.1).

EO 1100 further explicates: “Courses in subarea B4 shall have an explicit intermediate algebra prerequisite, and students shall develop skills and understanding beyond the level of intermediate algebra. Students will not just practice computational skills, but will be able to explain and apply basic mathematical concepts and will be able to solve problems through quantitative reasoning.”

§40402.1. Entry-Level Learning Skills.

Each student admitted to The California State University is expected to possess basic competence in the English language and mathematical computation to a degree reasonably expected of entering college students. Students admitted who cannot demonstrate such basic competence should be identified as quickly as possible and be required to take steps to overcome the deficiencies. Any coursework completed primarily for this purpose shall not be applicable to the baccalaureate degree.

Reference: §89030, California Education Code.

Attachments: Math Council Statement; GE Guiding Notes (excerpts on B4).

Approved unanimously—September 4, 2015

APPENDIX B: TASK FORCE MEMBERSHIP

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CSU Stanislaus

Katherine Stevenson
Professor of Mathematics
CSU Northridge

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Associate Professor of Mathematics
CSU San Marcos

Bill Eadie
Professor of Journalism & Media Studies
San Diego State University

Denise Fleming
Professor of Education
CSU East Bay

Catherine Nelson
Professor of Political Science
Sonoma State University

Mark Van Selst
Professor of Psychology
San José State University

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Deputy Superintendent
California Department of Education

Carolina Cardenas
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CSU Office of the Chancellor

Joey Freeman
Chief Policy Consultant
Lt. Governor's Office

Robyn Hines
Sr. Director, State Governmental Affairs
Microsoft

Monica Lin
Assoc. Director of Undergraduate Admissions
UC Office of the President

Gavin Newsom
Lieutenant Governor of California
State of California

Ken O'Donnell
Sr. Director, Student Engagement
CSU Office of the Chancellor

Ali Partovi
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Anaheim Union High School District

Ed Sullivan
Assistant Vice Chancellor,
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Steven Wood
Professor of Criminal Justice
CSU Stanislaus

APPENDIX C: COURSE AND ENROLLMENT DATA

The course and enrollment data cited in this report comes from these sources:

California High School Courses in Area c: Advanced Mathematics

Source: University of California Office of the President

Data Current as of April 15, 2016

California High School Courses in Area g: Electives with Quantitative Reasoning

Source: University of California Office of the President

Data Current as of June 14, 2016

California Community College Courses Approved for Transfer Credit in B4

Source: ASSIST Coordination Site, with invited corrections from colleges

Data Current as of June 17, 2016

California State University Courses in Area B4 of the GE Breadth Curriculum

Source: CSU Office of the Chancellor, with invited corrections from universities

Data Current as of June 17, 2016

The original records as provided to the Task Force are available for download in an Excel workbook, posted with this report under "Student Preparedness/Success" at calstate.edu/AcadSen/Records/Reports/index.shtml.

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APPENDIX E: ADDITIONAL RATIONALE FOR 12TH GRADE QUANTITATIVE REASONING

Not only is a fourth year of high school mathematics already recommended for all high school students intending to enroll in the CSU, but those students who are determined to be “conditionally ready” for college-level mathematics coursework are provided with an additional incentive to continue taking mathematics in their senior year of high school: By taking an approved senior-year math course and earning a grade of “C” or better, they do not need to participate in the Early Start summer program, nor will they need to take remedial mathematics courses at the CSU.

Students who take more mathematics in high school are less likely to need mathematics remediation. The College Board College-Bound Seniors Total Group Profile Reports [SAT 2013]–[SAT 2015] show that, year after year, the average SAT math score is less than 470 (33rd percentile) [WSAC 2014] for students who have only taken 3 years of high school, almost 520 (median) for students who have taken 4 years of high school mathematics, and over 570 (66th percentile) for students who have taken more than 4 years of high school mathematics. (For reference, the SAT score that the CSU accepts as indicating incoming proficiency in mathematics is 550.) ACT reports similar data [ACT 2007] with the percentage of students reaching the proficiency level (which ACT defines as a 22 on the ACT-Math test; note that the CSU threshold is a score of 23) more than doubled (from 16% to 38%) as the years of high school mathematics increased from 3 to 3.5, and increased almost fourfold (from 16% to 62%) as the years of high school mathematics increased from 3 to 4.

Students who take higher level math classes in high school are less likely to take a remedial mathematics course in college, one-third less likely according to [ACT 2007] if they have taken any advanced mathematics course after Algebra II. The Utah System of Higher Education reports that students who successfully completed a course beyond Algebra II were more than twice as likely to successfully meet the quantitative literacy requirement in college [USHE 2015].

Finally, the Quantitative Reasoning Task Force surveyed a number of public universities and university systems across the United States and found such requirements to be in existence in at least 21 states. The related links were accessed on June 16, 2016. As not every university was checked, there may be additional institutions with this same requirement that do not appear on the following list.

PUBLIC UNIVERSITIES AND SYSTEMS REQUIRING 4 YEARS OF HIGH SCHOOL
MATHEMATICS

ARIZONA

Arizona State University
students.asu.edu/freshman/requirements

Northern Arizona University
nau.edu/Admissions/Getting-Started/Requirements/Courses/

University of Arizona
admissions.arizona.edu/freshmen/entrance-requirements-and-guidelines

ARKANSAS

Arkansas State University
astate.edu/info/admissions/undergraduate/hs-core-curriculum/index.dot

University of Arkansas (Fayetteville)
admissions.uark.edu/apply/prepcore.php

University of Central Arkansas
uca.edu/admissions/apply/freshman/arkansased.gov/public/userfiles/Learning_Services/Curriculum_and_Instruction/Smartcore_Core/smartcore_course_2015_05142015.pdf

COLORADO

All four-year public institutions
higher.ed.colorado.gov/Academics/Admissions/coursecompletion.html
higher.ed.colorado.gov/Publications/Policies/Current/i-partf2019.Revise.pdf
colorado.edu/catalog/2015-16/content/minimum-academic-preparation-standards-maps
admissions.colostate.edu/18units/

FLORIDA

State University System of Florida
flbog.edu/documents_regulations/regulations/6_002_FTIC_Admissions_2_FINAL.pdf
admissions.ufl.edu/ugrad/frqualify.html

GEORGIA

University System of Georgia
usg.edu/assets/student_affairs/documents/Staying_on_Course.pdf

INDIANA

Purdue University System
admissions.purdue.edu/apply/highschoolcourses.php
admissions.purdue.edu/apply/mathcourses.php

LOUISIANA

Louisiana State University and A&M College (Baton Rouge)
sites01.lsu.edu/wp/admissions/become-a-tiger-2/freshmen/freshman-admission-requirements/

Southern University (Baton Rouge)
subr.edu/index.cfm/page/325/n/1524

University of New Orleans
uno.edu/admissions/freshman/academic-core-curriculum.aspx

MARYLAND

University System of Maryland
usmd.edu/newsroom/news/1021

Note: Beginning with the 9th grade class of fall 2014, the Maryland State Department of Education has required students to enroll in a mathematics course during each year of their high school career as a prerequisite for graduation.⁸

MASSACHUSETTS

Massachusetts State University System and University of Massachusetts System
mass.edu/shared/documents/admissions/admissionsstandards.pdf
bridgew.edu/admissions/undergraduate/apply
umass.edu/admissions/apply/admissions-requirements/freshman-admissions-requirements
umassd.edu/undergraduate/about/
uml.edu/admissions/freshmen-applicants.aspx

Note: The system-wide requirements take effect for students seeking admission in fall 2016. University of Massachusetts Amherst specifically requires students to take mathematics in the senior year.

MINNESOTA

University of Minnesota System
admissions.tc.umn.edu/counselors/math_requirement.html

Note: This requirement took effect for students seeking admission in fall 2015.

MISSOURI

University of Missouri System
umsystem.edu/ums/news/news_releases/um_enhances_admissions_policy
admissions.missouri.edu/apply/freshmen/requirements/high-school-coursework.php

NEBRASKA

University of Nebraska-Lincoln
admissions.unl.edu/apply.aspx#admission-requirements/freshmen

⁸See marylandpublicschools.org/programs/Pages/Testing/hs_gar.aspx#HSGR.

NEW MEXICO

University of New Mexico

admissions.unm.edu/future_students/admission-requirements.html

New Mexico State University

admissions.nmsu.edu/files/2015/11/2016-NMSU-Undergraduate-Viewbook.pdf

NORTH CAROLINA

University of North Carolina System

northcarolina.edu/prospective-students/minimum-admission-requirements

admissions.unc.edu/minimum-course-requirements/

SOUTH CAROLINA

All public senior colleges and universities colleges

[che.sc.gov/Portals/0/CHE_Docs/publications/AnnualReports/](http://che.sc.gov/Portals/0/CHE_Docs/publications/AnnualReports/Admissions_Standards_for_First-Time_Entering_Freshmen_FY2013-14.pdf)

[Admissions_Standards_for_First-Time_Entering_Freshmen_FY2013-14.pdf](http://che.sc.gov/Portals/0/CHE_Docs/publications/AnnualReports/Admissions_Standards_for_First-Time_Entering_Freshmen_FY2013-14.pdf)

che.sc.gov/CHE_Docs/AcademicAffairs/CollegePrepCourse.Prereqs101106.pdf

sc.edu/about/offices_and_divisions/undergraduate_admissions/requirements/

[for_freshmen/required_high_school_courses/index.php](http://sc.edu/about/offices_and_divisions/undergraduate_admissions/requirements/for_freshmen/required_high_school_courses/index.php)

scsu.edu/admissions/entrancerequirements/newfreshman.aspx

TENNESSEE

University of Tennessee at Chattanooga

utc.edu/admissions/apply/freshmanrequirements.php

University of Tennessee at Knoxville

admissions.utk.edu/apply/requirements/

University of Tennessee at Martin

utm.edu/departments/admissions/freshman.php

Note: The Tennessee Department of Education requires high schools students to earn four credits and to be enrolled in a mathematics course each year.⁹

TEXAS

The University of Texas at Austin

admissions.utexas.edu/explore/prerequisites/general-requirements

Texas A&M University (College Station)

admissions.tamu.edu/freshman/coursework

VIRGINIA

University of Virginia

admission.virginia.edu/admission

WEST VIRGINIA

University of West Virginia

admissions.wvu.edu/how-to-apply/first-time-freshmen#anchor-freshmanreqs

⁹ See tn.gov/education/topic/graduation-requirements.

WISCONSIN

University of Wisconsin-Madison

admissions.wisc.edu/apply/freshman/requirements.php

WYOMING

University of Wyoming

uwyo.edu/admissions/freshman/admissions-requirements.html

Additionally, some surveyed institutions, such as Indiana University Bloomington, require 3.5 years of high school mathematics.¹⁰ Others, such as Washington State University, require students to take a math-based quantitative course in their senior year of high school.¹¹

¹⁰See admissions.indiana.edu/apply/freshman/step-one.html.

¹¹For more information on Washington State University requirements, see: catalog.wsu.edu/General/AcademicRegulations/Search/both/admission; wsac.wa.gov/sites/default/files/2014.CADRS.Overview.pdf.

APPENDIX F: ACADEMIC SENATE CSU RESOLUTION 3253-16

Call for a Center for Advancement of Instruction in Mathematics

Resolved: That the Academic Senate of the California State University (ASCSU) encourage the establishment of a center to support mathematics instruction, analogous to the CSU Center for the Advancement of Reading (CAR); and be it further

Resolved: That the center have among its responsibilities:

- (a) development of a fourth-year high school mathematics course, analogous to the Expository Reading and Writing Course (ERWC);
 - (b) professional development for, and evaluation of, the fourth-year mathematics course;
 - (c) professional development in effective mathematics/quantitative reasoning instruction; and
 - (d) policy alignment in matters affecting mathematics curriculum and instruction;
- and be it further

Resolved: That the ASCSU distribute this resolution to the CSU Board of Trustees, CSU Chancellor, CSU campus Presidents, CSU campus Senate Chairs, CSU Provosts/Vice Presidents of Academic Affairs, CSU Math Council, CSU Deans of Colleges of Education, and the CSU Quantitative Reasoning Task Force.

Rationale. Currently, 27% of incoming CSU students arrive unprepared to succeed in college-level mathematics. In March 2016, the ASCSU passed AS-3244-16/APEP (Rev), "Support for Requiring a Fourth Year of Mathematics/Quantitative Reasoning for Admission to the California State University". Like the Center for the Advancement of Reading (CAR), this proposed center will provide leadership, support, training, and curricular resources in mathematics instruction for CSU faculty and California's K-12 teachers.

Approved unanimously — May 19–20, 2016