California State University Council of Math Chairs’
Statement on Entry Level Mathematics and Statway
30 April 2015

1. We support the Entry Level Mathematics (ELM) standards as the best measure of competency for entry to the CSU system. Students have multiple chances to meet these standards while in high school.

2. We request that the CSU Chancellor’s Office revise Executive Order 1065 (and 1100) so that all General Education Quantitative Reasoning courses have the content of ELM as an explicit prerequisite or co-requisite* and have explicit learning objectives that extend beyond ELM competency.

3. We encourage the CSU to focus on developing efforts such as Early Start (system-wide bridge courses for developmental math students that give them an extra preparation for their college level work). Our experience so far is that Early Start has the potential for substantially cutting the math remediation budget.

4. We oppose the exemption of Statway from Executive Order 1065. In order for Statway courses to meet the standards for transfer articulation with the CSU, they must have an explicit prerequisite or co-requisite* that subsumes the content of ELM, and the students’ ELM competency must be verified by proctored examinations.

5. We oppose the replacement of elementary or introductory statistics courses at CSU campuses by any program or pathway course lacking an explicit prerequisite or co-requisite* that subsumes the content of ELM. Such pathway courses include Statway. While the statistics content of Statway is totally aligned with the standard curriculum in elementary statistics, the pre-college mathematical content of Statway by itself does not meet the ELM standards and does not prepare students for college level courses. Hence Statway in its present form does not satisfactorily accomplish remediation and GE QR in a single track, thereby pointing to the need of having all ELM content in a prerequisite or co-requisite*.

*In any course with ELM content as a co-requisite, the students must meet preset competency levels on both the course’s measurable learning outcomes and on all the ELM topics, and these two sets of competencies must be separately assessed by proctored examinations. Students who pass the course proper but not its ELM co-requisite must undergo further remediation until their ELM competency reaches the preset level. At CSU campuses, Executive Order 665 must NOT be used to lift the ELM holds on such students.
Entry Level Mathematics (ELM): The standards and the exam

According to the CSU publication *Focus on Math*, the ELM placement exam has been used since 2002 to establish a student’s readiness for entry to the CSU system. The list of topics and example problems in *Focus on Math* and materials at several websites* establish specific competencies that a student should acquire before entering college level courses. These ELM standards have successfully served as the guidelines for Quantitative Reasoning readiness for the California Community Colleges, the CSU and the UC. The ELM standards are more accountable than the terms “intermediate algebra,” “remedial math” or “developmental math” since the meaning of these terms varies substantially. We find it useful to distinguish among the ELM *requirement* (which may be satisfied in several ways), the ELM *exam* (which is applied when a student has not met ELM requirements by other means), and the ELM *standards* (which state core topics and competencies).

*Websites related to the ELM requirement, ELM exam, and ELM standards:
https://www.csumathsuccess.org/elm_requirement
http://study.com/academy/course/elm-test.html
https://www.ets.org/csu/about/elm/elm_topics

ELM is Important for a Well Informed Citizenry

The ELM standards set a foundation for understanding today’s world that is as relevant as standards for critical reading and writing. ELM competency --- including sound evaluation of graphs, statistics and numerical information --- is vital to an informed citizenry. The ability to understand graphs, translate socially relevant challenges into mathematical models, manipulate formulas to perform calculations, and restate the outcomes as solutions to the said challenges, are considered basic requirements in almost all entry-level positions for college graduates. CSU developmental math courses teach to the ELM standards. Students learn how to read graphs, build math models, manage unknowns, solve linear & quadratic equations, and justify answers. The courses aim to cultivate fluency in navigating among the tabular, graphical, algebraic, and contextualized representations of data. While the immediate value of ELM competency is its role in preparing students for GE level and upper-division Quantitative Reasoning courses, its full value is probably not assessable during their tenure on campus. Students will be tapping into that literacy for life and throughout their careers.
The following table links several topics from the ELM standards with practical issues.

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<tr>
<th>Content areas</th>
<th>Topics</th>
<th>Social relevance</th>
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<tbody>
<tr>
<td><strong>Data &amp; Numbers (~35%)</strong></td>
<td>Representation of data (tables, pie charts, histograms, graphs, etc.); basic probability (mean, median, variance); estimates and predictions. Arithmetic; percentages, fractions, decimals; ratios &amp; proportions. Estimation (of square roots, etc.).</td>
<td>Making responsible decisions. Analyzing various scientific &amp; financial situations. Understanding graphs in social science. Navigating through tax forms. Figuring out insurance premiums. Adjusting cooking recipes and mixing up compound products (drugs, food mixes). Prioritizing the use of multiple discount coupons to one’s best advantage. Developing a good sense about orders of magnitude.</td>
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<td><strong>Algebra (~35%)</strong></td>
<td>Linear equations &amp; inequalities (single unknown or systems with two unknowns); slopes &amp; intercepts. Quadratic* functions. Average rates and rational expressions. Arithmetic for simplifying algebraic expressions. Equations &amp; inequalities with absolute values. Properties of exponents.**</td>
<td>Choosing wisely among several vacation packages or job offers. Performing simple revenue-profit analyses when the number of sales depends linearly on the price. Appreciating the effects of key parameters behind projectile motion. Calculating the amount of land needed for a preset crop-yield. Navigation in the presence of currents. Understanding basic models in physics. Arm-chair astronomy. Finding the sample size needed for any specific margin of error. Average costs. Elasticity of demand (comparing the percentage change of demand to that of price). Basic spreadsheet analysis skills such as supply &amp; demand projections.***</td>
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<td><strong>Geometry (~30%)</strong></td>
<td>Perimeter, area, volume of various geometrical objects; how the ratio between perimeters transforms to ratios between areas and between volumes. Properties of congruent /similar shapes. Pythagoras’s Theorem. Concept of angles. Intersecting, parallel, or perpendicular lines. Plotting points on the number line and in the coordinate plane. Length &amp; midpoint of line segments. Graphing linear, quadratic, and algebraic functions; relating the geometrical features of the graphs to the formulae of the functions.</td>
<td>Evaluating designs &amp; the aesthetics of symmetry. Developing a good sense about proportions and similarity. Reading blueprints and interpreting architectural drawings, and applying those to carpentry. Exploiting the scaling properties of areas and volumes in making cost-efficient decisions. Ability to navigate fluently among four manifestations of data: graphs, tables or charts, formulae, and socially relevant contexts.</td>
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*For details about the socially relevance of quadratic functions cited in this article, see: plus.math.org/content/101-uses-quadratic-equation; plus.math.org/content/101-uses-quadratic-equation-part-ii; mathsisfun.com/algebra/quadratic-equation-real-world.html.

**Some developmental math curricula include exponential functions in order to distinguish them from powers of x, thereby leading naturally to logarithms. This literacy is important for everyday life and many GE Science courses. It concerns mortgages (the magic of extra principal reduction, choosing among refinance options, etc.), compound interest issues (such as the approximate number of years it takes to double one’s investment, present & future values, inflation), decibels in acoustics, pitch in music, the Richter scale, pH in chemistry, human perception graphs in psychology, spectrometry, noise, image compression, growth models, fractal dimension, complexity, chaos and entropy, etc.

*** An example of basic supply & demand projections: Estimating the number of seats needed in a course, based on the attrition rate of students taking the course, and the success rates at which students are fulfilling the prerequisites for that course.
Remediation and Innovative Approaches at CSU campuses

Throughout the system, approximately 50% of entering freshman have satisfied the ELM requirement. As for the rest, over 70% complete remediation within their first year. Some campuses such as Channel Islands, Dominguez Hills, and Los Angeles, have even higher pass rates. By the end of their first year, well over 85% of entering freshman are ELM compliant/exempt. Early Start Mathematics and periodic curriculum enrichments have further improved the pass rates.

The math departments of many CSU campuses have already been working as a group towards redesigns of the traditional pathway (consisting of remediation followed by GE Quantitative Reasoning courses) that can be shared across all campuses. The Math Council embraces pedagogical innovations that maintain the ELM content as well as its contextualized approach. The Math Chairs, on behalf of their departments, welcome the opportunity to work with the Chancellors Office to continue these innovations and develop methods to efficiently scale them for broader use in the system.

**Applications just-in-time**  At CSU campuses, ELM content is being taught with immediate applications to socially relevant contexts. We do not subscribe to the model of teaching only theory and telling the students that they will encounter applications in later courses. Furthermore, once the tools have been introduced and exemplified, students are being asked to practice, practice, and more practice. This is one reason the pass rates are so high.

**Harnessing the potential of ESM (Early Start Math)**  Brick & mortar ESM classes have impressive pass rates, so most students get to move up to the next remedial math course or into GE QR during the first term of their freshman year. Alternatively, one could invest serious effort in online ESM and then strongly encourage the hard-working students to retake the ELM exam in August.

**Humboldt’s 5-unit fast track**  The traditional pathway towards completing GE QR, for students needing only 1 semester of remediation, is MATH 44, 103, in succession, for a total of 6 units over 2 semesters. As an alternative, students with majors outside of the College of Natural Resources & Sciences can take 2 units of MATH 43 concurrently with 3 units of MATH 103i, for a total of 5 units in 1 semester. The pass rate for the 5-unit 1-semester pathway is superior to that for the traditional 6-unit 2-semester pathway.

**Statpath at Northridge**  For students needing only one semester of remediation, namely those with ELM scores 34-48, Statpath provides an alternative: 5 units of developmental math with a special curriculum, followed by 3 units of traditional GE stats. In that special curriculum, all material is presented in context and some time is spent on pre-stats content instead of rational expressions. Though there is no savings in total units or semesters compared to the traditional pathway, Statpath aims to provide a better bridge from ELM to GE stats, thereby reducing the number of D or F or W grades in the latter.

**Enriched curriculum for developmental math at CSU campuses**  The developmental math programs at many (as of this writing, at least 9) CSU campuses have undergone substantial overhaul & enrichment, in order to better align with Common Core. At some campuses such as SF State, the redesign was carried out with an eye towards a smoother transition into GE QR courses. For example, pre-stats content eases the transition into elementary stats, and a solid introduction of exponentials & logarithms helps students who will be taking pre-calculus.

**Supplementary workshops or labs**  These are optional 1-unit classes that are run by students (graduate and/or undergraduate). Currently, such workshops are companion to pre-calculus & calculus classes. There are structured activities to engage the students, and the latter do get one-on-one help. Statistics have shown that veterans of such workshops have better study skills and better track records throughout their undergraduate careers than the typical students. This paradigm helped CSU Monterey Bay improve its developmental math program and is being used at several community colleges in the Los Angeles area.
What is Statway?

Statway is a proprietary curriculum for elementary statistics, designed by the Carnegie Foundation. It is inquiry based, and is intended for small classes of 20-30 students due to its reliance on group work. The syllabus for a standard one-term elementary statistics course is stretched to two terms. The first term typically covers combinatorics & probability, leading to a statement of the Central Limit Theorem. The second term covers confidence intervals & hypothesis testing, with the option of including goodness-of-fit at the end. Statway does not require intermediate algebra or ELM competency as a prerequisite, and limits its coverage of Entry Level Math to arithmetic and straight lines (& the option of covering a bit of exponential functions), with the latter done in the context of regression.

Proponents of Statway say that traditional intermediate algebra goes way beyond the minimal level of mathematical competency for college readiness. They feel that many topics in intermediate algebra are primarily about the mechanics of manipulating polynomials, rational functions, exponentials & logarithms, and as such are only relevant to STEM-bound students. These proponents also claim that Statway sufficiently covers mathematics that is aligned with the Common Core State Standards.

CSU Council of Math Chairs’ Position on Statway

ELM competency should be the standard for college readiness and lifelong numeracy. Enriched curricula that teach the ELM standards should contain only a small amount of polynomial mechanics, and (except for the absence of rudimentary probability) should be completely aligned with the topics stipulated in the Common Core State Standards for Mathematics. As such, they lay a quantitative foundation for most of the college curriculum. In contrast, the current version of Statway, though well aligned with the standard curriculum in elementary statistics, only covers the very limited amount of ELM content that is necessary for learning statistics; hence Statway’s coverage of the Common Core State Standards in Math is insufficient for the subsequent coursework needs of its veterans. Students who took Statway at community colleges and transfer to CSU may have to be remediated on-the-fly in order to meet their subsequent needs at CSU campuses, incurring hidden costs and lengthening their time to graduation.

Regardless of where Statway is taught, as a GE Quantitative Reasoning course, at the community colleges or on CSU campuses, the Math Council insists that the ELM topics should comprise a prerequisite or a co-requisite, and the students’ ELM competency should be verified by proctored examinations. Independent tracking needs to be carried out for Statway veterans’ performance in subsequent CSU courses. The onus is on the Carnegie Foundation to prove to the CSU that Statway does work; namely, that Statway veterans are ELM competent and do as well in subsequent courses as their counterparts in the traditional pathway.

Since its inquiry-based curriculum is designed for small classes (20-30 students), Statway doesn’t scale. Due to budgetary reasons, elementary statistics at some CSU campuses are taught in large sections of 100 or more students. Many university students called to ask why they were not allowed to enroll in the small Statway sections and some students have protested that it is discriminatory to limit those sections to a select few (e.g. at San Francisco State, there were 75 Statway students among a total of 1200 students in elementary statistics).
CSU’s experience with Statway

The Chancellors Office persuaded five CSU campuses to offer Statway. The following table summarizes their experiences.

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<th>Pilot campuses</th>
<th>East Bay (EB), Northridge, Sacramento (SAC), San Francisco (SF), and San Jose (SJ). Northridge opted out from the get-go; SF has opted out as of AY 2014-15; EB will follow suit in AY 2015-16.</th>
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<tr>
<td>Paradigms</td>
<td>EB, SAC, SF, have piloted enriched Statway, in which most of the missing ELM content has been restored via a parallel track. SJ teaches essentially plain Statway.</td>
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<td>Students at East Bay and San Francisco State were required to sign up for additional units. All of the instructors reported anxiety and frustration stemming from the challenges of teaching two parallel tracks, and the compensation was not commensurate with the amount of effort invested.</td>
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<td>ELM score ranges and restrictions of students in the pilots</td>
<td>30-40 at EB; 44-48 non-STEM &amp; non-BUS at SAC; 0-40 Metro Health Academy cohort at SF; 0-42 with EPT higher than 139 at SJ, taught out of Undergrad Studies instead.</td>
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<td>ELM competency of SW veterans</td>
<td>EB: On a developmental math common final, traditional students averaged 23% points higher than Statway students. 51% of traditional students scored at least 60/100, versus 5% of Statway students.</td>
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<td>As for course grades in elementary statistics, 61% of traditional students received a course grade of C or higher, while 95% of Statway students received at least a B.</td>
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<td>SAC</td>
<td>Since SW veterans at SAC State begin with ELM scores 44-48, their ELM competency after completing Statway is less of a concern than that at the other three CSU campuses (EB 30-40, SF 0-40, SJ 0-42). A simple diagnostic would be to have these Statway veterans retake the ELM exam or some equivalent test (such as the Intermediate Algebra Diagnostic).</td>
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<td>SF</td>
<td>Statway veterans have weaker fluency among the graphical, tabular, algebraic, and contextualised manifestations of quantitative data, even though the three instructors invested thrice as much effort!</td>
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<td></td>
<td>What if the ELM content were not taught in a parallel track at SFSU? Consider, say, linear equations. Veterans of ELM would have been holistically drilled on those four manifestations. On the other hand, the Statway curriculum without ELM retrofit only covers linear relationships in the context of regression analysis, with most of the mechanics done by the computer. The latter exposure, being much narrower, would exacerbate the said weakness.</td>
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<td>SJ</td>
<td>Instructors lauded the merits of Statway and the approach of supplementing the basic algebra &amp; arithmetic when needed. But one instructor candidly lamented that “the algebra instruction is not even good enough for the statistics we need to do”.</td>
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After piloting Statway at several CSU campuses, we conclude that the pre-college mathematical content of Statway by itself does not meet the ELM standards and does not prepare students for college level courses, hence it is necessary to impose on Statway an explicit prerequisite or co-requisite that subsumes the content of ELM.